Strong planning

In this chapter, we will consider the simplest case of nondeterministic planning by restricting attention to strong plans.

Strong plans

Recall the definition of strong plans:

Definition (strong plan)
Let $S$ be the set of states of a planning task $\Pi$. Then a strong plan for $\Pi$ is a function $\pi : S_\pi \rightarrow O$ for some subset $S_\pi \subseteq S$ such that

- $\pi(s)$ is applicable in $s$ for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi \cup S_*$ ($\pi$ is closed),
- $S_\pi(s') \cap S_* \neq \emptyset$ for all $s' \in S_\pi(s_0)$ ($\pi$ is proper), and
- there is no state $s' \in S_\pi(s_0)$ such that $s'$ is reachable from $s'$ following $\pi$ in a strictly positive number of steps ($\pi$ is acyclic).
**Strong plans**

**Execution of a strong plan**

1. Determine the current state $s$.
2. If $s$ is a goal state then terminate.
3. Execute action $\pi(s)$.
4. Repeat from first step.

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**Images**

The **image** of a set $T$ of states with respect to an operator $o$ is the set of those states that can be reached by executing $o$ in a state in $T$.

$$img_o(T) = \{ s' \in S | s \xrightarrow{o} s' \} = app_o(s)$$

**Definition (image of a state)**

$$img_o(s) = \{ s' \in S | s \xrightarrow{o} s' \} = app_o(s)$$

**Definition (image of a set of states)**

$$img_o(T) = \bigcup_{s \in T} img_o(s)$$
Weak preimages

**Weak preimage**

The **weak preimage** of a set \( T \) of states with respect to an operator \( o \) is the set of those states from which a state in \( T \) can be reached by executing \( o \).

\[
\text{wpreimg}_o(T) = \{ s \in S | s \xrightarrow{o} s' \} 
\]

Strong preimages

**Strong preimage**

The **strong preimage** of a set \( T \) of states with respect to an operator \( o \) is the set of those states from which a state in \( T \) is always reached when executing \( o \).

\[
\text{spreimg}_o(T) = \{ s \in S | \exists s' \in T : s \xrightarrow{o} s' \land \text{img}_o(s) \subseteq T \}
\]
Algorithms

Dynamic programming

Planning by dynamic programming
If for all successors of state $s$ with respect to operator $o$ a plan exists, assign operator $o$ to $s$.
- **Base case** \( i = 0 \): In goal states there is nothing to do.
- **Inductive case** \( i \geq 1 \): If $\pi(s)$ is still undefined and there is $o \in O$ such that for all $s' \in \text{img}_o(s)$, the state $s'$ is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

Backward distances
If $s$ is assigned a value on iteration $i \geq 1$, then the **backward distance** of $s$ is $i$. The dynamic programming algorithm essentially computes the backward distances of states.

Algorithms for strong planning

1. **Dynamic programming** (backward)
   - Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.
   - Zero actions needed for goal states.
   - If states with $i$ actions to goals are known, states with $\leq i + 1$ actions to goals can be easily identified.
   - Automatic reuse of plan suffixes already found.

2. **Heuristic search** (forward)
   - Strong planning can be viewed as AND/OR graph search.
   - OR nodes: Choice between operators
   - AND nodes: Choice between effects
   - Heuristic AND/OR search algorithms: AO*, Proof Number Search, …
Definition (backward distance sets)
Let $G$ be a set of states and $O$ a set of operators. The backward distance sets $D_{bwd}^i$ for $G$ and $O$ consist of those states for which there is a guarantee of reaching a state in $G$ with at most $i$ operator applications using operators in $O$:

$$D_{bwd}^0 := G$$
$$D_{bwd}^i := D_{bwd}^{i-1} \cup \bigcup_{o \in O} \text{sprimg}_o(D_{bwd}^{i-1})$$

for all $i \geq 1$.

Strong plans based on distances
Let $\Pi = (V, I, O, \gamma)$ be a nondeterministic planning task with state set $S$ and goal states $S^\ast$. Extraction of a strong plan from distance sets
1. Let $S' \subseteq S$ be those states having a finite backward distance for $G = S^\ast$ and $O$.
2. Let $s \in S'$ be a state with distance $i = \delta_{bwd}^G(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $\text{img}_o(s) \subseteq D_{bwd}^{i-1}$. Hence $o$ decreases the backward distance by at least one.

Then $\pi$ is a strong plan for $T$ iff $I \in S'$.

Question: What is the worst-case runtime of the algorithm?
We have considered the special case of nondeterministic planning where
- planning tasks are fully observable and
- we are interested in strong plans.

We have introduced important concepts also relevant to other variants of nondeterministic planning such as
- images and
- weak and strong preimages.

We have discussed one basic classes of algorithms:
backward induction by dynamic programming.