In this chapter, we will consider the simplest case of nondeterministic planning by restricting attention to strong plans.
Concepts
Recall the definition of strong plans:

**Definition (strong plan)**

Let $S$ be the set of states of a planning task $\Pi$. Then a strong plan for $\Pi$ is a function $\pi : S_\pi \rightarrow O$ for some subset $S_\pi \subseteq S$ such that

- $\pi(s)$ is applicable in $s$ for all $s \in S_\pi$,
- $S_\pi(s_0) \subseteq S_\pi \cup S_\pi^\ast$ ($\pi$ is closed),
- $S_\pi(s') \cap S_\pi^\ast \neq \emptyset$ for all $s' \in S_\pi(s_0)$ ($\pi$ is proper), and
- there is no state $s' \in S_\pi(s_0)$ such that $s'$ is reachable from $s'$ following $\pi$ in a strictly positive number of steps ($\pi$ is acyclic).
Strong plans

Execution of a strong plan

1. Determine the current state $s$.
2. If $s$ is a goal state then terminate.
3. Execute action $\pi(s)$.
4. Repeat from first step.
Strong plans

- Strong plans
- Images
- Weak preimages
- Strong preimages

Algorithms

Summary

Images

Image

The image of a set $T$ of states with respect to an operator $o$ is the set of those states that can be reached by executing $o$ in a state in $T$. 

$$\text{img}_o(T)$$
Definitions

**Definition (image of a state)**

\[ \text{img}_o(s) = \{ s' \in S | s \xrightarrow{o} s' \} = \text{app}_o(s) \]

**Definition (image of a set of states)**

\[ \text{img}_o(T) = \bigcup_{s \in T} \text{img}_o(s) \]
Weak preimages

Weak preimage

The weak preimage of a set \( T \) of states with respect to an operator \( o \) is the set of those states from which a state in \( T \) can be reached by executing \( o \).
Weak preimages

Definition (weak preimage of a state)

\[ \text{wpreimg}_o(s') = \{ s \in S | s \xrightarrow{0} s' \} \]

Definition (weak preimage of a set of states)

\[ \text{wpreimg}_o(T) = \bigcup_{s \in T} \text{wpreimg}_o(s). \]
Strong preimage

The **strong preimage** of a set \( T \) of states with respect to an operator \( o \) is the set of those states from which a state in \( T \) is always reached when executing \( o \).

\[
spreimg_o(T)
\]

\( T \)
Strong preimages

Definition (strong preimage of a set of states)

\[ \text{spreimg}_o(T) = \{ s \in S \mid \exists s' \in T : s \xrightarrow{\circ} s' \land \text{img}_o(s) \subseteq T \} \]
Algorithms
Algorithms for strong planning

1 Dynamic programming (backward)
Compute operator/distance/value for a state based on the operators/distances/values of its all successor states.

- Zero actions needed for goal states.
- If states with \(i\) actions to goals are known, states with \(\leq i + 1\) actions to goals can be easily identified.

Automatic reuse of plan suffixes already found.

2 Heuristic search (forward)
Strong planning can be viewed as AND/OR graph search.

- OR nodes: Choice between operators
- AND nodes: Choice between effects

Heuristic AND/OR search algorithms:
\( AO^* \), Proof Number Search, ...
Dynamic programming

Planning by dynamic programming

If for all successors of state $s$ with respect to operator $o$ a plan exists, assign operator $o$ to $s$.

- **Base case $i = 0$**: In goal states there is nothing to do.
- **Inductive case $i \geq 1$**: If $\pi(s)$ is still undefined and there is $o \in O$ such that for all $s' \in \text{img}_o(s)$, the state $s'$ is a goal state or $\pi(s')$ was assigned in an earlier iteration, then assign $\pi(s) = o$.

Backward distances

If $s$ is assigned a value on iteration $i \geq 1$, then the **backward distance** of $s$ is $i$. The dynamic programming algorithm essentially computes the **backward distances** of states.
Backward distances

Example

distance to $G$

$\infty$ 3 2 1 0

$G$
Backward distances

Definition (backward distance sets)

Let $G$ be a set of states and $O$ a set of operators. The backward distance sets $D_i^{\text{bwd}}$ for $G$ and $O$ consist of those states for which there is a guarantee of reaching a state in $G$ with at most $i$ operator applications using operators in $O$:

$$
D_0^{\text{bwd}} := G
$$

$$
D_i^{\text{bwd}} := D_{i-1}^{\text{bwd}} \cup \bigcup_{o \in O} \text{spreimg}_o(D_{i-1}^{\text{bwd}}) \quad \text{for all } i \geq 1
$$
Definition (backward distance)

Let $G$ be a set of states and $O$ a set of operators, and let $D_{bwd}^0, D_{bwd}^1, \ldots$ be the backward distance sets for $G$ and $O$. Then the backward distance of a state $s$ for $G$ and $O$ is

$$
\delta_{bwd}^G(s) = \min\{i \in \mathbb{N} \mid s \in D_{bwd}^i\}
$$

(where $\min\emptyset = \infty$).
Concepts
Algorithms
Regression
Summary

Strong plans based on distances

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set $S$ and goal states $S_\star$.

Extraction of a strong plan from distance sets

1. Let $S' \subseteq S$ be those states having a finite backward distance for $G = S_\star$ and $O$.
2. Let $s \in S'$ be a state with distance $i = \delta_{G}^{bwd}(s) \geq 1$.
3. Assign to $\pi(s)$ any operator $o \in O$ such that $\text{img}_o(s) \subseteq D_{i-1}^{bwd}$. Hence $o$ decreases the backward distance by at least one.

Then $\pi$ is a strong plan for $\mathcal{T}$ iff $I \in S'$.

Question: What is the worst-case runtime of the algorithm?
Summary
Summary

- We have considered the special case of nondeterministic planning where
  - planning tasks are fully observable and
  - we are interested in strong plans.

- We have introduced important concepts also relevant to other variants of nondeterministic planning such as
  - images and
  - weak and strong preimages.

- We have discussed one basic classes of algorithms: backward induction by dynamic programming.