Exercise 12.1 (BDDs, 2+2+2 points)

Let $\varphi$ be the formula $(v_1 \land v_2) \lor (v_3 \land \neg v_4)$ over the variables $A = \{v_1, \ldots, v_4\}$, and let $S$ be the set of states 0010, 0011, 0100, 0110, 1000, 1011, 1100, 1110 and 1111 over $A$, where $xyzu$ is a shorthand for $\{v_1 \mapsto x, v_2 \mapsto y, v_3 \mapsto z, v_4 \mapsto u\}$. Moreover, let $B$ be the BDD over the variables $v_1, \ldots, v_5$ depicted on the right.

(a) Reduce $B$ as much as possible. Give intermediate results after each reduction step. Give representations of $B$ as a formula in conjunctive normal form and as a set of states.

(b) Represent $\varphi$ as a reduced ordered BDD and as a set of states.

(c) Represent $S$ as a reduced ordered BDD and as a formula in conjunctive normal form.

Exercise 12.2 (Operators as BDDs, 4 points)

Let $A = \{a, b, c\}$ and let $o = \langle \top, c \rangle$ be an operator over $A$. Specify the reduced ordered BDDs corresponding to the (sequential) encoding $\tau_A(o)$ for the variable orderings $a \prec b \prec c \prec a' \prec b' \prec c'$ and $a \prec a' \prec b \prec b' \prec c \prec c'$ on $A \cup A'$.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.