Principles of AI Planning

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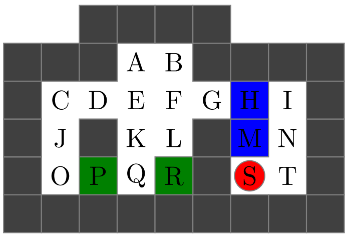
Exercise Sheet 9

Due: Friday, December 22nd, 2017

Exercise 9.1 (Additive patterns and canonical heuristic, 3+1+2 points)

Consider the Sokoban problem given by the picture below. The red circle denotes the agent's position, the blue squares are boxes, and the green grid cells are the target positions of the boxes (it is irrelevant which box ends up in which target position). The letters only denote the names of the grid cells.



We will model this problem in finite-domain representation using the variables $\mathit{position}_p$, $\mathit{position}_{s_1}$, $\mathit{position}_{s_2}$, $\mathit{at\text{-}goal}_{s_1}$, $\mathit{at\text{-}goal}_{s_2}$, $\mathit{content}_A$, $\mathit{content}_B, \ldots, \mathit{content}_T$ with the following domains:

- $\mathcal{D}_{\mathit{position}_p} = \mathcal{D}_{\mathit{position}_{s_1}} = \mathcal{D}_{\mathit{position}_{s_2}} = \{A, B, \ldots, T\}$
- $\mathcal{D}_{\mathit{at\text{-}goal}_{s_1}} = \mathcal{D}_{\mathit{at\text{-}goal}_{s_2}} = \{\text{true}, \text{false}\}$
- $\mathcal{D}_{\mathit{content}_A} = \cdots = \mathcal{D}_{\mathit{content}_T} = \{\text{nothing}, p, s_1, s_2\}$

The initial state is given as

- $\mathit{position}_p = S, \mathit{position}_{s_1} = M, \mathit{position}_{s_2} = H, \mathit{at\text{-}goal}_{s_1} = \mathit{at\text{-}goal}_{s_2} = \text{false}$
- $\mathit{content}_H = s_2, \mathit{content}_M = s_1, \mathit{content}_S = p$
- $\mathit{content}_X = \text{nothing for } X \in \{A, \ldots, T\} \setminus \{H, M, S\}$

and the goal formula is $\mathit{at\text{-}goal}_{s_1} = \text{true} \land \mathit{at\text{-}goal}_{s_2} = \text{true}$. The set of available operators contains the obvious FDR formalizations of all *move*- and *push*-actions that are usually available in Sokoban.

Consider the pattern collection $\mathcal{C}$ with the following patterns:

$$
\begin{aligned}
P_1 &= \{\mathit{at\text{-}goal}_{s_2}\} \\
P_2 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{position}_{s_1}\} \\
P_3 &= \{\mathit{at\text{-}goal}_{s_2}, \mathit{position}_{s_2}\} \\
P_4 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{position}_{s_1}, \mathit{position}_p\} \\
P_5 &= \{\mathit{position}_{s_1}, \mathit{position}_p\} \\
P_6 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_H\} \\
P_7 &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_G\} \\
P_8 &= \{\mathit{at\text{-}goal}_{s_2}, \mathit{content}_D\} \\
P_9 &= \{\mathit{content}_A, \mathit{content}_E\} \\
P_{10} &= \{\mathit{at\text{-}goal}_{s_1}, \mathit{content}_Q\}
\end{aligned}
$$

(a) Specify the compatibility graph of $\mathcal{C}$ and determine its maximal cliques.

(b) Determine the canonical heuristic $h^{\mathcal{C}}$.

(c) Not all patterns in $\mathcal{C}$ are reasonable. Which can obviously be omitted, and why? What would the canonical heuristic look like if we omitted those patterns before even constructing the compatibility graph?

Exercise 9.2 (Accuracy of pattern database heuristics, 4 points)

Show that PDB heuristics can become arbitrarily inaccurate even if almost all variables are represented in the pattern.

More formally: Show that there exists a family of FDR planning tasks $(\Pi_n)_{n\in\mathbb{N}}$ (that are not trivially unsolvable and that contain no trivially inapplicable operators) with $\Pi_n = (V_n, I_n, O_n, \gamma_n)$ where $|V_n| = \Theta(n)$, $h^*(I_n) = \Omega(n)$, and such that for all patterns $P_n \subsetneq V_n$ with $|P_n| = |V_n| - 1$, we have $h^{P_n}(I_n) = O(1)$.

Hint: Generalize the logistics example from slides 7-9 in `aip11.pdf`.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.