Exercise 4.1 ($A^*$ search, 3+3 points)
Assume that you have a puzzle consisting of five cells. The first two cells contain black tiles, the next two white ones, and the last cell is empty.

A tile can be moved into a neighbored empty cell (costing one unit) or a tile can “jump” over at most two cells into an empty cell (costing the number of cells jumped over). The goal of the game is to have both black tiles to the right of the white tiles, while the empty cell may have an arbitrary position. Note that unlike in the lecture, we have non-unit action costs here, but the generalization of $A^*$ search to this setting is obvious ($g(s)$ now denotes the sum of action costs so far instead of the number of actions so far).

(a) Solve the puzzle with the $A^*$ algorithm and the following heuristic function $h$: Each white tile to the right of the first black tile costs one unit, and each white tile to the right of the second black tile costs one unit. E.g., $h(I) = 4$ for the initial state $I$ depicted above.

(b) Show that $h$ is admissible, i.e., $h(s) \leq h^*(s)$ for all states $s$.

Exercise 4.2 (Enforced hill-climbing, 4 points)
Hansel and Gretel are lost in a rectangular forest. Even worse, they are separated from each other. Fortunately, there is the friendly neighborhood witch who is willing to help them reunite. As an appropriate place for the meeting she proposes her gingerbread house in the middle of the forest. Hansel and Gretel can both move in four directions, namely north, east, south and west. Call the actions $north_H$, $south_H$, $east_H$, $west_H$, $north_G$, $south_G$, $east_G$, and $west_G$, respectively, with the obvious semantics of moving either Hansel or Gretel by one grid cell in the indicated direction. Some cells with extremely thick vegetation are inaccessible (shown in grey in the figure below) and cannot be moved to. The goal is to move both Hansel and Gretel to the goal position in the center of the grid, marked by $X$. Their current positions are indicated by $H$ and $G$, respectively.

Solve the problem with the enforced hill-climbing algorithm. A planning state is uniquely identified by the coordinates of Hansel and Gretel. To estimate the goal distance of a state, use the sum of distances of both Hansel and Gretel to the goal position in east-west and north-south directions. E.g., the heuristic value for the initial state $I$ is $h(I) = |3-1| + |3-2| + |3-4| = 5$. Whenever tie-breaking is necessary in the algorithm, you may choose the most suitable step first. For each invocation of the $improve$ procedure, specify the state after the improvement by giving the new coordinates of Hansel and Gretel. Finally, record the solution plan.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.