

Principles of AI Planning

15. Nondeterministic planning

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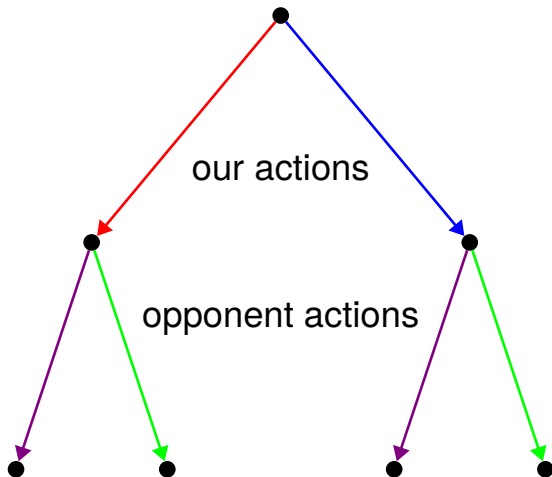
- The world is not predictable.
- AI robotics:
 - imprecise movement of the robot
 - other robots
 - human beings, animals
 - machines (cars, trains, airplanes, lawn-mowers, ...)
 - natural phenomena (wind, water, snow, temperature, ...)
- Games: other players are outside our control.
 - To win a game (reaching a goal state) with certainty, all possible actions by the other players have to be anticipated (a **winning strategy** of a game).
 - The world is not predictable because it is unknown: we cannot **observe** everything.

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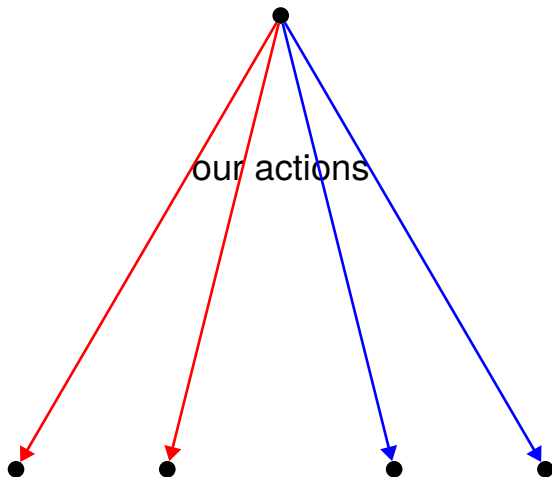
In this lecture, we will only deal with uncertain operator outcomes, not with partial observability.



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- In **deterministic planning** we have assumed that the only changes taking place in the world are those caused by us and that we can **exactly predict** the results of our actions.
- **Other agents** and processes, beyond our control, are formalized as **nondeterminism**.
- Implications:
 - 1 The future state of the world cannot be predicted.
 - 2 We cannot reliably plan ahead: no single operator sequence achieves the goals.
 - 3 In some cases it is not possible to achieve the goals with certainty no matter which outcomes the actions have, but only under certain fairness assumptions.

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Transition systems with nondeterminism (cf. Chapter 2)

Definition (transition system)

A **nondeterministic transition system** is a 5-tuple

$\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ where

- S is a finite set of **states**,
- L is a finite set of (transition) **labels**,
- $T \subseteq S \times L \times S$ is the **transition relation**,
- $s_0 \in S$ is the **initial state**, and
- $S_\star \subseteq S$ is the set of **goal states**.

Note: $T \subseteq S \times L \times S$ allows **nondeterministic operators** with more than one possible outcome.

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Definition (nondeterministic operator)

Let V be a set of finite-domain state variables. A nondeterministic operator in unary nondeterminism normal form with conjunctive precondition and unconditional effects, or **nondeterministic operator** for short, is a pair $o = \langle \chi, E \rangle$, where

- χ is a conjunction of atoms over V (the **precondition**), and
- $E = \{e_1, \dots, e_n\}$ is a finite set of possible **effects** of o , each e_i being a conjunction of atomic finite-domain effects over V .

Definition (nondeterministic operator application)

Let $o = \langle \chi, E \rangle$ be a nondeterministic operator and s a state.

Applicability of o in s is defined as in the deterministic case, i.e., o is **applicable** in s iff $s \models \chi$ and the change set of each effect $e \in E$ is consistent.

If o is applicable in s , then the **application** of o in s leads to one of the states in the set $app_o(s) := \{app_{\langle \chi, e \rangle}(s) \mid e \in E\}$ nondeterministically.

Example

$put-on-block(A, B) = \langle \chi, \{e_1, e_2\} \rangle$ where

- $\chi = \{handempty \mapsto false, clear-B \mapsto true, pos-A \mapsto hand\},$
- $e_1 = \{handempty \mapsto true, clear-B \mapsto false, pos-A \mapsto on-B\},$
- $e_2 = \{handempty \mapsto true, pos-A \mapsto table\}.$

Applied to a state where the agent is holding block A and block B is clear, this operator leads to one of two possible successor states. Either A gets stacked on B successfully, or A is dropped to the table.

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Definition (nondeterministic planning task)

A (fully observable) **nondeterministic planning task** is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- V is a finite set of **finite-domain state variables**,
- I is an **initial state** over V ,
- O is a finite set of **nondeterministic operators** over V , and
- γ is a conjunctions of atoms over V describing the **goal states**.

Remark: In the following, we will always assume that our nondeterministic planning tasks are fully observable.

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Definition (induced transition system)

Every nondeterministic planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces a corresponding nondeterministic transition system

$$\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle:$$

- S is the set of all states over V ,
- L is the set of operators O ,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' \in \text{app}_o(s) \},$
- $s_0 = I$, and
- $S_\star = \{ s \in S \mid s \models \gamma \}$



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Definition

What is a plan?

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In nondeterministic planning, plans are more complicated objects than in the deterministic case:

The best action to take may **depend on nondeterministic effects** of previous operators.

Nondeterministic plans thus often require **branching**.
Sometimes, they even require **looping**.

Example (Branching)

(Part of) a plan for winning the game **Connect Four** can be described as follows:

- Place a tile in the 4th column.
 - If opponent places a tile in the 1st, 4th or 7th column, place a tile in the 4th column.
 - If opponent places a tile in the 2nd or 5th column, place a tile in the 2nd column.
 - If opponent places a tile in the 3rd or 6th column, place a tile in the 6th column.

There is no **non-branching** plan that solves the task
(= is guaranteed to win the game).

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What is a plan?

Example (Looping)

A plan for building a card house can be described as follows:

- 1 Build a wall with two cards.
If the structure falls apart, redo from start.
- 2 Build a second wall with two cards.
If the structure falls apart, redo from start.
- 3 Build a ceiling on top of the walls with a fifth card.
If the structure falls apart, redo from start.
- 4 Build a wall on top of the ceiling with two cards.
If the structure falls apart, redo from start.

There is no **non-looping** plan that solves the task
(unless the planning agent is very dextrous).

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What is a plan?

- Plans should be allowed to **branch**. Otherwise, most interesting nondeterministic planning tasks cannot be solved.
- We may or may not allow plans to **loop**.
 - Non-looping plans are preferable because they **guarantee** that the goal is reached within a bounded number of steps.
 - Where non-looping plans are not possible, looping plans may be adequate because they at least guarantee that the goal will be reached **eventually** unless nature is **unfair**.

We will now introduce the formal concepts necessary to define branching and looping plans.

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Definition (strategy)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states S_* .

A **strategy** for Π is a function $\pi : S_\pi \rightarrow O$ for some subset $S_\pi \subseteq S$ such that for all states $s \in S_\pi$ the action $\pi(s)$ is applicable in s .

The set of states reachable in $\mathcal{T}(\Pi)$ starting in state s and following π is denoted by $S_\pi(s)$.

Definition (weak, closed, proper, and acyclic strategies)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states S_* , and let π be a strategy for Π .

Then π is called

- **weak** iff $S_\pi(s_0) \cap S_* \neq \emptyset$,
- **closed** iff $S_\pi(s_0) \subseteq S_\pi \cup S_*$,
- **proper** iff $S_\pi(s') \cap S_* \neq \emptyset$ for all $s' \in S_\pi(s_0)$, and
- **acyclic** iff there is no state $s' \in S_\pi(s_0)$ such that s' is reachable from s' following π in a strictly positive number of steps.

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- **Strategies** in nondeterministic planning correspond to **applicable operator sequences** in deterministic planning.
- In deterministic planning, a **plan** is an applicable operator sequence that results in a goal state.
- In nondeterministic planning, we define different notions of “resulting in a goal state”.

Definition

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a nondeterministic planning task with state set S and goal states S_* .

- A strategy for Π is called a **weak plan** for Π iff it is weak.
- A strategy for Π is called a **strong cyclic plan** for Π iff it is closed and proper.
- A strong cyclic plan for Π is called a **strong plan** for Π iff it is acyclic.

We extended the deterministic (**classical**) planning formalism:

- **operators** can be nondeterministic

Remark: We could also introduce nondeterminism in the initial situation by allowing more than one initial state, but this can be easily compiled into our formalism. (**How?**)

As a consequence, **plans** can contain

- **branches** and
- **loops**.

In the following chapter, we consider the **strong planning** problem and the **strong cyclic planning** problem and discuss some algorithms.

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