Principles of AI Planning

14. Planning as search: Partial-Order Reduction

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Motivation

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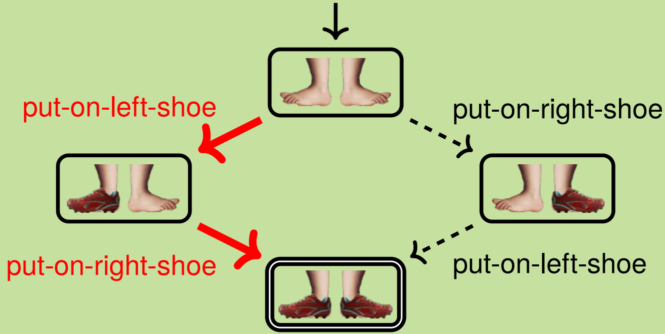
- Worst case: Heuristic search may explore exponentially more states than necessary, even if heuristic is almost perfect (Helmert and Röger, 2008).
- Example: A* search in GRIPPER domain explores all permutations of ball transportations if heuristic is off only by a small constant.
- Idea: Complement heuristic search with orthogonal technique(s) to reduce size of explored state space.
- Desired properties of this technique: preservation of completeness and, if possible, optimality.

Idea:

- Enforce particular ordering among operators.
- Ignore all other orderings.

Example



Preliminaries

Setting

Assumption: For the rest of the chapter, we assume that all planning tasks are SAS$^+$ planning tasks $\Pi = (V, I, O, \gamma)$.

For convenience, we assume that operators have the form $o = \langle pre(o), eff(o) \rangle$, where $pre(o)$ and $eff(o)$ are both partial states over V, i.e., partial functions mapping variables v to values in $\mathscr{D}_v$. Similarly, we assume that γ is a partial state describing the goal.

Example

Operator $o = \langle pre(o), eff(o) \rangle$ with

- $pre(o) = \{v_1 \mapsto d_1, v_5 \mapsto d_5\}$ and
- $eff(o) = \{v_2 \mapsto d_2, v_3 \mapsto d_3\}$

corresponds to $o = \langle \chi, e \rangle$ with
$\chi = (v_1 = d_1 \wedge v_5 = d_5)$ and $e = (v_2 := d_2 \wedge v_3 := d_3)$.

Definition (Operators)

Let $\Pi = (V, I, O, \gamma)$ be a SAS$^+$ planning task and $o \in O$ an operator. Then

- $\mathit{prevars}(o) := \mathit{vars}(\mathit{pre}(o))$ are the variables that occur in the precondition of o.
- $\mathit{effvars}(o) := \mathit{vars}(\mathit{eff}(o))$ are the variables that occur in the effect of o.
- o reads $v \in V$ iff $v \in \mathit{prevars}(o)$.
- o modifies $v \in V$ iff $v \in \mathit{effvars}(o)$.

Variable $v \in V$ is goal-related iff $v \in \mathit{vars}(\gamma)$.

Assumption: $\mathit{effvars}(o) \neq \emptyset$ for all $o \in O$.

Operator Dependencies

Definition (Operator dependencies)

Let $\Pi = \langle V, O, I, \gamma \rangle$ be a planning task and $o, o' \in O$.

1. o disables o' iff there exists $v \in \mathit{effvars}(o) \cap \mathit{prevars}(o')$ such that $\mathit{eff}(o)(v) \neq \mathit{pre}(o')(v)$.
2. o enables o' iff there exists $v \in \mathit{effvars}(o) \cap \mathit{prevars}(o')$ such that $\mathit{eff}(o)(v) = \mathit{pre}(o')(v)$.
3. o and o' conflict iff there is $v \in \mathit{effvars}(o) \cap \mathit{effvars}(o')$ such that $\mathit{eff}(o)(v) \neq \mathit{eff}(o')(v)$.
4. o and o' interfere iff o disables o', or o' disables o, or o and o' conflict.
5. o and o' are commutative iff o and o' do not interfere, and neither o enables o', nor o' enables o.

Operator Dependencies

Example

$$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$$
$$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$$
$$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$$
$$\text{go-to-gym} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{gym} \rangle$$

Then:

- go-to-uni and go-to-gym disable put-on-left and put-on-right.
- put-on-left and put-on-right enable go-to-uni and go-to-gym.
- go-to-uni and go-to-gym conflict.
- put-on-left and put-on-right are commutative.

Necessary Enabling Sets and Disjunctive Action Landmarks

Definition (Necessary enabling set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task, s a state, and $o \in O$ an operator that is not applicable in s. A set N of operators is a necessary enabling set (NES) for o in s if all operator sequences that lead from s to a goal state and include o contain an operator in N before the first occurrence of o.

Note: NESs not uniquely determined for given o and s.
(E.g., supersets of NESs are still NESs.)

Necessary Enabling Sets and Disjunctive Action Landmarks

Definition (Disjunctive action landmark)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A disjunctive action landmark (DAL) L in s is a set of operators such that all operator sequences that lead from s to a goal state contain some operator in L.

Observation

For state s and operator o that is not applicable in s, disjunctive action landmarks for task $\langle V, I, O, pre(o) \rangle$ are necessary enabling sets for o in s.

Necessary Enabling Sets and Disjunctive Action Landmarks

Proof

Let L be such a disjunctive action landmark.

Then each operator sequence that leads from s to a state satisfying $pre(o)$ contains some operator in L.

Thus, each operator sequence that leads from s to a goal state and includes o contains an operator in L before the first occurrence of o.

Therefore, L is an NES for o in s.

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Stubborn Sets

Stubborn Sets

Back to the motivation:

If, in state s, some set of operators can be applied in any order and the order does not matter, we want to commit to one such order and ignore all other orders.

Idea:

Identify operators that can be postponed since they are independent of all operators that are not postponed.

E.g., put-on-right could be postponed, since it is independent of put-on-left (that is not postponed).

Stubborn Sets

Idea (more precisely): Identify operators that should not be postponed, and postpone the rest.

Question: When should an operator o not be postponed?

Answer:

1. Base case: If o may be immediately relevant to reaching (part of) the goal, or
2. Inductive case I: If o may be immediately relevant to contributing to making another operator applicable that should not be postponed, or
3. Inductive case II: If o might not be applicable any more if we postponed it, or if its effect might conflict with the effect of another operator that should not be postponed ($\approx$ o interferes with such an operator).

Strong Stubborn Sets

Let's formalize the above answer:

Definition (Strong stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a strong stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains all operators that interfere with o.

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s.

Strong Stubborn Sets

Example

$$I = \{\text{pos} \mapsto \text{home}, \text{left} \mapsto \text{f}, \text{right} \mapsto \text{f}\}, \quad \gamma = \{\text{pos} \mapsto \text{uni}\}$$

$$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$$

$$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$$

$$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$$

- Step 1: DAL in I is $\{\text{go-to-uni}\} \rightsquigarrow T_s := \{\text{go-to-uni}\}$.
- Step 2: go-to-uni not applicable in I. One possible NES for go-to-uni in I is $\{\text{put-on-left}\} \rightsquigarrow T_s := T_s \cup \{\text{put-on-left}\}$.
- Step 3: put-on-left is applicable in I. The only operator that interferes with it, go-to-uni, is already in T_s.
- Hence, $T_s = \{\text{go-to-uni}, \text{put-on-left}\}$, and T_s restricted to the applicable operators is $\{\text{put-on-left}\}$. During search, only apply put-on-left (not put-on-right).

Strong Stubborn Sets

Example

Let $V = \{u_1, u_2, v, w\}$, $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$, $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$, and $O = \{o_1, o_2, o_3\}$, where:

- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2\rangle$,
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2\rangle$,
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1\rangle$.

Strong stubborn set:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Include o_3 in T_s since it interferes with o_1 (or o_2).
- Step 3: Include o_2 (or o_1) in T_s since it interferes with o_3.

$\leadsto$ all applicable operators included in T_s, no pruning.

Question: Can we do better than that in this example?

Domain Transition Graphs

Definition (Domain transition graph)

Let $\Pi = (V, I, O, \gamma)$ be a SAS$^+$ planning task and $v \in V$. The domain transition graph for v is the directed graph $DTG(v) = \langle \mathscr{D}_v, E \rangle$ where $(d, d') \in E$ iff there is an operator $o \in O$ with

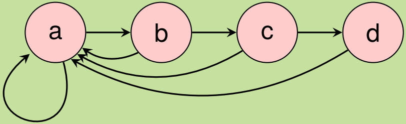
- $\mathit{eff}(o)(v) = d'$, and
- $v \notin \mathit{prevars}(o)$ or $\mathit{pre}(o)(v) = d$.

Domain Transition Graphs

Example

$$\text{move-a-b} = \langle \text{pos} = \text{a}, \text{pos} := \text{b} \rangle$$
$$\text{move-b-c} = \langle \text{pos} = \text{b}, \text{pos} := \text{c} \rangle$$
$$\text{move-c-d} = \langle \text{pos} = \text{c}, \text{pos} := \text{d} \rangle$$
$$\text{reset} = \langle \top, \text{pos} := \text{a} \wedge \text{othervar} := \text{otherval} \rangle$$

Then *DTG*(pos):



Active Operators

Definition (Active operators)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and let s be a state. The set of active operators $Act(s) \subseteq O$ in s is defined as the set of operators such that for all $o \in Act(s)$:

- For every variable $v \in prevars(o)$, there is a path in $DTG(v)$ from $s(v)$ to $pre(o)(v)$. If v is goal-related, then there is also a path from $pre(o)(v)$ to the goal value $\gamma(v)$.
- For every goal-related variable $v \in effvars(o)$, there is a path in $DTG(v)$ from $eff(o)(v)$ to the goal value $\gamma(v)$.

Active Operators

Proposition

1. $Act(s)$ can be identified efficiently for a given state s by considering paths in the projection of Π onto v.
2. Operators not in $Act(s)$ can be treated as nonexistent when reasoning about s because they are not applicable in all states reachable from s, or they lead to a dead-end from s.

Proof

1. Homework: Specify efficient algorithm for identification of $Act(s)$.
2. Obvious. $\square$

Active Operators

Remark 1: Even when excluding inactive operators, this preserves completeness and even optimality of a search algorithm (see proof below).

Remark 2: Excluding inactive operators can "cascade" in the sense that additional active operators need not be considered.

Definition (Strong stubborn set with active operator pruning)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a strong stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains all operators that are active in s and interfere with o.

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s.

Strong Stubborn Sets

Recall the previous example where strong stubborn sets without active operator pruning were useless.

Example

- $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$,
  $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$
- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$

Now, with active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state.
  $\rightsquigarrow$ o_3 not in T_s, although it interferes with o_1 (or o_2).

Example (Example, ctd.)

Now, with active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state. $\rightsquigarrow$ o_3 not in T_s, although it interferes with o_1 (or o_2).
- Hence, e. g., $T_s = \{o_1\}$ strong stubborn set (with active operator pruning) in I.
- Even active operator o_2 is not included in $T_s = \{o_1\}$.

$\rightsquigarrow$ some pruning occurs.

Weak Stubborn Sets

With weak stubborn sets, some operators that disable an operator in T_s need not be included in T_s.

Therefore, weak stubborn sets potentially allow more pruning than strong stubborn sets.

Definition (Weak stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a weak stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains the active operators in s that have conflicting effects with o or that are disabled by o.

Weak Stubborn Sets

For weak stubborn sets, it suffices to include active operators o' that are disabled or conflict with applicable operators $o \in T_s$. However, o' does not need to be included if o' disables an applicable operator $o \in T_s$.

No computational overhead of computing weak stubborn sets over computing strong stubborn sets.

Theorem

In the best case, weak stubborn sets admit exponentially more pruning than strong stubborn sets.

Proof

Homework. □

Algorithms

compute-DAL: Compute a disjunctive action landmark.

Precedure compute-DAL

def compute-DAL(γ):
  select $v \in vars(\gamma)$ with $s(v) \neq \gamma(v)$
  $L \leftarrow \{o' \in Act(s) \mid eff(o')(v) = \gamma(v)\}$
  return L

Selection of $v \in vars(\gamma)$ arbitrary. Any variable will do.
Selection heuristics?

compute-NES: Compute a necessary enabling set.

Precedure compute-NES

def compute-NES(o,s):
  select $v \in prevars(o)$ with $s(v) \neq pre(o)(v)$
  $N \leftarrow \{o' \in Act(s) \mid eff(o')(v) = pre(o)(v)\}$
  return N

Selection of $v \in prevars(o)$ arbitrary. Any variable will do.
Selection heuristics?

Algorithms

compute-interfering-operators: Compute interfering operators.

Precedure compute-interfering-operators (for strong SS)

def compute-interfering-operators(o):
  disablers $\leftarrow \{o' \in O \mid o'$ disables $o\}$
  disablees $\leftarrow \{o' \in O \mid o$ disables $o'\}$
  conflicting $\leftarrow \{o' \in O \mid o$ and o' conflict$\}$
  return disablers $\cup$ disablees $\cup$ conflicting

Precedure compute-interfering-operators (for weak SS)

def compute-interfering-operators(o):
  disablees $\leftarrow \{o' \in O \mid o$ disables $o'\}$
  conflicting $\leftarrow \{o' \in O \mid o$ and o' conflict$\}$
  return disablees $\cup$ conflicting

Algorithms

Computing (strong and weak) stubborn sets for planning can be achieved with a fixpoint iteration until the constraints of T_s are satisfied:

compute-stubborn-set: Compute (strong or weak) stubborn set.

Precedure compute-stubborn-set

def compute-stubborn-set(s):
  $T_s \leftarrow$ compute-DAL(γ)
  while no fixed-point of T_s reached **do**
    for $o \in T_s$ applicable in s:
      $T_s \leftarrow T_s \cup$ compute-interfering-operators(o)
    for $o \in T_s$ not applicable in s:
      $T_s \leftarrow T_s \cup$ compute-NES(o, s)
  end while
  return T_s

Integration into A*

Observation: stubborn sets are state-dependent, but not path-dependent.

This allows filtering the applicable operators in *s* in graph search algorithms like A* that perform duplicate detection, too.

Instead of applying all applicable operators $app(s)$ in s, only apply operators in $T_{app(s)} := T_s \cap app(s)$.

Theorem

Weak stubborn sets are completeness and optimality preserving.

Proof

Let $T_{app(s)} := T_s \cap app(s)$ for a weak stubborn set T_s.

We show that for all states s from which an optimal plan consisting of $n > 0$ operators exists, $T_{app(s)}$ contains an operator that starts such a plan.

We show by induction that A* restricting successor generation to $T_{app(s)}$ is optimal.

Let T_s be a weak stubborn set and $\pi = o_1, \ldots, o_n$ be an optimal plan that starts in s.

...

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Preservation of Completeness and Optimality

Proof (ctd.)

As T_s contains a disjunctive action landmark, π must contain an operator from T_s.

Let o_k be the operator with smallest index in π that is also contained in T_s, i.e., $o_k \in T_s$ and $\{o_1, \dots, o_{k-1}\} \cap T_s = \emptyset$.

We observe:

1. $o_k \in app(s)$: otherwise by definition of weak stubborn sets, a necessary enabling set N for o_k in s would have to be contained in T_s, and at least one operator from N would have to occur before o_k in π to enable o_k, contradicting that o_k was chosen with smallest index.
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2. ...

Preservation of Completeness and Optimality

Proof (ctd.)

1. …
2. o_k is does not disable any of the operators $o_1, \dots, o_{k-1}$, and all these operators have non-conflicting effects with o_k: otherwise, as $o_k \in app(s)$, and by definition of weak stubborn sets, at least one of $o_1, \dots, o_{k-1}$ would have to be contained in T_s, again contradicting the assumption.

Hence, we can move o_k to the front:
$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$ is also a plan for Π.
It has the same cost as π and is hence optimal.
Thus, we have found an optimal plan of length n started by an operator $o_k \in T_{app(s)}$, completing the proof. □

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It has the same cost as π and is hence optimal.

Thus, we have found an optimal plan of length n started by an operator $o_k \in T_{app(s)}$, completing the proof. □

Remark: The argument to move o_k to the front also holds for strong stubborn sets: in this case, o_k is not even disabled by any of $o_1, \ldots, o_{k-1}$ (and hence, o_k is independent of $o_1, \ldots, o_{k-1}$), which is a stronger property than needed in the proof.

Corollary

Strong stubborn sets are completeness and optimality preserving. □

Some Experiments: Overview

Optimal Planning, A* with LM-cut Heuristic, Selected Domains

Domain (problems)	Coverage A*	Coverage +SSS	Nodes generated A*	Nodes generated +SSS
PARCPRINTER-08 (30)	18	**+12**	2455181	<**1%**
PARCPRINTER-OPT11 (20)	13	**+7**	2454533	<**1%**
WOODWORKING-OPT08 (30)	17	**+10**	26796212	<**1%**
WOODWORKING-OPT11 (20)	12	**+7**	26795517	<**1%**
SATELLITE (36)	7	**+5**	5116312	**2%**
ROVERS (40)	7	**+2**	1900691	**22%**
AIRPORT (50)	**28**	±**0**	545072	**93%**
OPENSTACKS-OPT08 (30)	19	**+2**	56584063	**51%**
OPENSTACKS-OPT11 (20)	14	**+2**	56456969	**51%**
DRIVERLOG (20)	13	**+1**	3679376	**82%**
SCANALYZER-08 (30)	**15**	−3	**14203012**	**100%**
SCANALYZER-OPT11 (20)	**12**	−3	**14202884**	**100%**
PARKING-OPT11 (20)	**3**	−1	**560914**	**100%**
SOKOBAN-OPT08 (30)	**30**	−1	**20519270**	**100%**
VISITALL-OPT11 (20)	**11**	−1	**1991169**	**100%**
REMAINING DOMAINS (980)	**544**	±**0**	436017004	**93%**
SUM (1396)	763	**+39**	670278179	**77%**

Domain (problems)	Coverage SSS	Coverage WSS	Nodes generated SSS	Nodes generated WSS	# problems w. diff. gen.
OPENSTACKS-OPT08 (30)	**21**	$\pm$**0**	152711917	**99.936%**	18
OPENSTACKS-OPT11 (20)	**16**	$\pm$**0**	152642101	**99.936%**	16
PATHWAYS-NONEG (30)	**5**	$\pm$**0**	162347	**99.702%**	2
PSR-SMALL (50)	**49**	$\pm$**0**	18119489	**99.998%**	6
SATELLITE (36)	**12**	$\pm$**0**	70299721	**92.804%**	12

$\Rightarrow$ In practice (or, at least, in the standard benchmark problems) there is no significant difference between weak and strong stubborn sets.

Conclusion

Conclusion

- Need for techniques orthogonal to heuristic search, complementing heuristics.
- One idea: Commit to one order of operators if they are independent. Prune other orders.
- Class of such techniques: partial-order reduction (POR)
- One such technique: strong/weak stubborn sets
- Can lead to substantial pruning compared to plain A*.
- Many other POR techniques exist.
- Other pruning techniques exist as well, e.g., symmetry reduction.