Principles of AI Planning

14. Planning as search: Partial-Order Reduction

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Motivation

Motivation

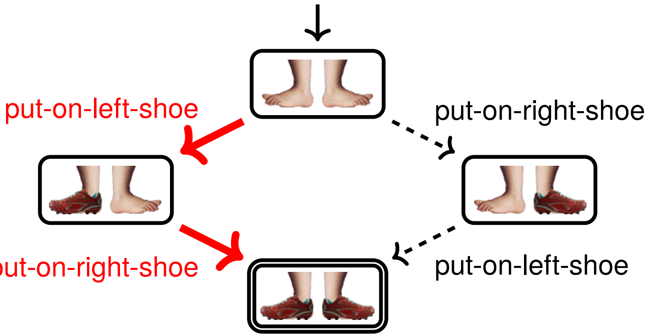
- Worst case: Heuristic search may explore exponentially more states than necessary, even if heuristic is almost perfect (Helmert and Röger, 2008).
- Example: A* search in GRIPPER domain explores all permutations of ball transportations if heuristic is off only by a small constant.
- Idea: Complement heuristic search with orthogonal technique(s) to reduce size of explored state space.
- Desired properties of this technique: preservation of completeness and, if possible, optimality.

Partial-Order Reduction

Idea:

- Enforce particular ordering among operators.
- Ignore all other orderings.

Example



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Preliminaries

Setting

Assumption: For the rest of the chapter, we assume that all planning tasks are SAS$^+$ planning tasks $\Pi = (V, I, O, \gamma)$.

For convenience, we assume that operators have the form $o = \langle pre(o), eff(o) \rangle$, where $pre(o)$ and $eff(o)$ are both partial states over V, i.e., partial functions mapping variables v to values in $\mathscr{D}_v$. Similarly, we assume that γ is a partial state describing the goal.

Example

Operator $o = \langle pre(o), eff(o) \rangle$ with

- $pre(o) = \{v_1 \mapsto d_1, v_5 \mapsto d_5\}$ and
- $eff(o) = \{v_2 \mapsto d_2, v_3 \mapsto d_3\}$

corresponds to $o = \langle \chi, e \rangle$ with
$\chi = (v_1 = d_1 \land v_5 = d_5)$ and $e = (v_2 := d_2 \land v_3 := d_3)$.

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Basic Definitions

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Definition (Operators)

Let $\Pi = (V, I, O, \gamma)$ be a SAS$^+$ planning task and $o \in O$ an operator. Then

- $\mathit{prevars}(o) := \mathit{vars}(\mathit{pre}(o))$ are the variables that occur in the precondition of o.
- $\mathit{effvars}(o) := \mathit{vars}(\mathit{eff}(o))$ are the variables that occur in the effect of o.
- o reads $v \in V$ iff $v \in \mathit{prevars}(o)$.
- o modifies $v \in V$ iff $v \in \mathit{effvars}(o)$.

Variable $v \in V$ is goal-related iff $v \in \mathit{vars}(\gamma)$.

Assumption: $\mathit{effvars}(o) \neq \emptyset$ for all $o \in O$.

Operator Dependencies

Definition (Operator dependencies)

Let $\Pi = \langle V, O, I, \gamma \rangle$ be a planning task and $o, o' \in O$.

1. o disables o' iff there exists $v \in \mathit{effvars}(o) \cap \mathit{prevars}(o')$ such that $\mathit{eff}(o)(v) \neq \mathit{pre}(o')(v)$.
2. o enables o' iff there exists $v \in \mathit{effvars}(o) \cap \mathit{prevars}(o')$ such that $\mathit{eff}(o)(v) = \mathit{pre}(o')(v)$.
3. o and o' conflict iff there is $v \in \mathit{effvars}(o) \cap \mathit{effvars}(o')$ such that $\mathit{eff}(o)(v) \neq \mathit{eff}(o')(v)$.
4. o and o' interfere iff o disables o', or o' disables o, or o and o' conflict.
5. o and o' are commutative iff o and o' do not interfere, and neither o enables o', nor o' enables o.

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Operator Dependencies

Example

$$\begin{aligned}
\text{put-on-left} &= \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle \\
\text{put-on-right} &= \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle \\
\text{go-to-uni} &= \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle \\
\text{go-to-gym} &= \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{gym} \rangle
\end{aligned}$$

Then:

- go-to-uni and go-to-gym disable put-on-left and put-on-right.
- put-on-left and put-on-right enable go-to-uni and go-to-gym.
- go-to-uni and go-to-gym conflict.
- put-on-left and put-on-right are commutative.

Necessary Enabling Sets and Disjunctive Action Landmarks

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Definition (Necessary enabling set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task, s a state, and $o \in O$ an operator that is not applicable in s. A set N of operators is a necessary enabling set (NES) for o in s if all operator sequences that lead from s to a goal state and include o contain an operator in N before the first occurrence of o.

Note: NESs not uniquely determined for given o and s. (E.g., supersets of NESs are still NESs.)

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Definition (Disjunctive action landmark)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A disjunctive action landmark (DAL) L in s is a set of operators such that all operator sequences that lead from s to a goal state contain some operator in L.

Observation

For state s and operator o that is not applicable in s, disjunctive action landmarks for task $\langle V, I, O, pre(o) \rangle$ are necessary enabling sets for o in s.

Necessary Enabling Sets and Disjunctive Action Landmarks

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Proof

Let L be such a disjunctive action landmark.

Then each operator sequence that leads from s to a state satisfying $\mathit{pre}(o)$ contains some operator in L.

Thus, each operator sequence that leads from s to a goal state and includes o contains an operator in L before the first occurrence of o.

Therefore, L is an NES for o in s.

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Stubborn Sets

Stubborn Sets

Back to the motivation:

If, in state *s*, some set of operators can be applied in any order and the order does not matter, we want to commit to one such order and ignore all other orders.

Idea:

Identify operators that can be postponed since they are independent of all operators that are not postponed.

E.g., put-on-right could be postponed, since it is independent of put-on-left (that is not postponed).

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Idea (more precisely): Identify operators that should not be postponed, and postpone the rest.

Question: When should an operator *o* not be postponed?

Answer:

1. Base case: If *o* may be immediately relevant to reaching (part of) the goal, or
2. Inductive case I: If *o* may be immediately relevant to contributing to making another operator applicable that should not be postponed, or
3. Inductive case II: If *o* might not be applicable any more if we postponed it, or if its effect might conflict with the effect of another operator that should not be postponed ($\approx$ *o* interferes with such an operator).

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Strong Stubborn Sets

Let's formalize the above answer:

Definition (Strong stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a strong stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains all operators that interfere with o.

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s.

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Example

$$I = \{\text{pos} \mapsto \text{home}, \text{left} \mapsto \text{f}, \text{right} \mapsto \text{f}\}, \quad \gamma = \{\text{pos} \mapsto \text{uni}\}$$
$$\text{put-on-left} = \langle \text{pos} = \text{home} \wedge \text{left} = \text{f}, \text{left} := \text{t} \rangle$$
$$\text{put-on-right} = \langle \text{pos} = \text{home} \wedge \text{right} = \text{f}, \text{right} := \text{t} \rangle$$
$$\text{go-to-uni} = \langle \text{left} = \text{t} \wedge \text{right} = \text{t}, \text{pos} := \text{uni} \rangle$$

- Step 1: DAL in I is $\{\text{go-to-uni}\} \rightsquigarrow T_s := \{\text{go-to-uni}\}$.
- Step 2: go-to-uni not applicable in I. One possible NES for go-to-uni in I is $\{\text{put-on-left}\} \rightsquigarrow T_s := T_s \cup \{\text{put-on-left}\}$.
- Step 3: put-on-left is applicable in I. The only operator that interferes with it, go-to-uni, is already in T_s.
- Hence, $T_s = \{\text{go-to-uni}, \text{put-on-left}\}$, and T_s restricted to the applicable operators is $\{\text{put-on-left}\}$. During search, only apply put-on-left (not put-on-right).

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Example

Let $V = \{u_1, u_2, v, w\}$, $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$, $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$, and $O = \{o_1, o_2, o_3\}$, where:

- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$,
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$,
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$.

Strong stubborn set:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Include o_3 in T_s since it interferes with o_1 (or o_2).
- Step 3: Include o_2 (or o_1) in T_s since it interferes with o_3.

$\rightsquigarrow$ all applicable operators included in T_s, no pruning.

Question: Can we do better than that in this example?

Domain Transition Graphs

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Definition (Domain transition graph)

Let $\Pi = (V, I, O, \gamma)$ be a SAS$^+$ planning task and $v \in V$. The domain transition graph for v is the directed graph $DTG(v) = \langle \mathscr{D}_v, E \rangle$ where $(d, d') \in E$ iff there is an operator $o \in O$ with

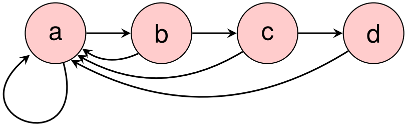
- $\mathit{eff}(o)(v) = d'$, and
- $v \notin \mathit{prevars}(o)$ or $\mathit{pre}(o)(v) = d$.

Domain Transition Graphs

Example

$$\begin{aligned}
\text{move-a-b} &= \langle \text{pos} = \text{a}, \text{pos} := \text{b} \rangle \\
\text{move-b-c} &= \langle \text{pos} = \text{b}, \text{pos} := \text{c} \rangle \\
\text{move-c-d} &= \langle \text{pos} = \text{c}, \text{pos} := \text{d} \rangle \\
\text{reset} &= \langle \top, \text{pos} := \text{a} \wedge \text{othervar} := \text{otherval} \rangle
\end{aligned}$$

Then *DTG*(pos):



$$\begin{tikzcd}
a \arrow[r] \arrow[loop below] & b \arrow[r] \arrow[l, bend left] & c \arrow[r] \arrow[ll, bend left] & d \arrow[lll, bend left]
\end{tikzcd}$$

Active Operators

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Definition (Active operators)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and let s be a state. The set of active operators $Act(s) \subseteq O$ in s is defined as the set of operators such that for all $o \in Act(s)$:

- For every variable $v \in prevars(o)$, there is a path in $DTG(v)$ from $s(v)$ to $pre(o)(v)$. If v is goal-related, then there is also a path from $pre(o)(v)$ to the goal value $\gamma(v)$.
- For every goal-related variable $v \in effvars(o)$, there is a path in $DTG(v)$ from $eff(o)(v)$ to the goal value $\gamma(v)$.

Active Operators

Proposition

1. $Act(s)$ can be identified efficiently for a given state s by considering paths in the projection of Π onto v.
2. Operators not in $Act(s)$ can be treated as nonexistent when reasoning about s because they are not applicable in all states reachable from s, or they lead to a dead-end from s.

Proof

1. Homework: Specify efficient algorithm for identification of $Act(s)$.
2. Obvious. $\square$

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Active Operators

Remark 1: Even when excluding inactive operators, this preserves completeness and even optimality of a search algorithm (see proof below).

Remark 2: Excluding inactive operators can "cascade" in the sense that additional active operators need not be considered.

Strong Stubborn Sets

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Definition (Strong stubborn set with active operator pruning)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a strong stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains all operators that are active in s and interfere with o.

Instead of applying all applicable operators in s only apply those that are applicable and contained in T_s.

Strong Stubborn Sets

Why operator activity matters

Recall the previous example where strong stubborn sets without active operator pruning were useless.

Example

- $I = \{u_1 \mapsto 0, u_2 \mapsto 0, v \mapsto 0, w \mapsto 0\}$,
  $\gamma = \{v \mapsto 0, u_1 \mapsto 1, u_2 \mapsto 1\}$
- $o_1 = \langle u_1 = 0, u_1 := 1 \wedge w := 2 \rangle$
- $o_2 = \langle u_2 = 0, u_2 := 1 \wedge w := 2 \rangle$
- $o_3 = \langle u_1 = 0 \wedge u_2 = 0, v := 1 \wedge w := 1 \rangle$

Now, with active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state.
  $\rightsquigarrow$ o_3 not in T_s, although it interferes with o_1 (or o_2).

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Strong Stubborn Sets

Why operator activity matters

Example (Example, ctd.)

Now, with active operator pruning:

- Step 1: Include o_1 (or o_2) in T_s as DAL.
- Step 2: Operator o_3 is not active in any reachable state.
  $\rightsquigarrow$ o_3 not in T_s, although it interferes with o_1 (or o_2).
- Hence, e. g., $T_s = \{o_1\}$ strong stubborn set (with active operator pruning) in I.
- Even active operator o_2 is not included in $T_s = \{o_1\}$.

$\rightsquigarrow$ some pruning occurs.

Weak Stubborn Sets

With weak stubborn sets, some operators that disable an operator in T_s need not be included in T_s.

Therefore, weak stubborn sets potentially allow more pruning than strong stubborn sets.

Definition (Weak stubborn set)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a planning task and s a state. A set $T_s \subseteq O$ is a weak stubborn set in s if

1. T_s contains a disjunctive action landmark in s, and
2. for all $o \in T_s$ that are not applicable in s, T_s contains a necessary enabling set for o and s, and
3. for all $o \in T_s$ that are applicable in s, T_s contains the active operators in s that have conflicting effects with o or that are disabled by o.

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Weak Stubborn Sets

For weak stubborn sets, it suffices to include active operators o' that are disabled or conflict with applicable operators $o \in T_s$. However, o' does not need to be included if o' disables an applicable operator $o \in T_s$.

No computational overhead of computing weak stubborn sets over computing strong stubborn sets.

Theorem

In the best case, weak stubborn sets admit exponentially more pruning than strong stubborn sets.

Proof

Homework. $\square$

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compute-DAL: Compute a disjunctive action landmark.

Precedure compute-DAL

```
def compute-DAL($\gamma$):
    select $v \in vars(\gamma)$ with $s(v) \neq \gamma(v)$
    $L \leftarrow \{o' \in Act(s) \mid eff(o')(v) = \gamma(v)\}$
    return $L$
```

Selection of $v \in vars(\gamma)$ arbitrary. Any variable will do.
Selection heuristics?

Algorithms

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compute-NES: Compute a necessary enabling set.

Precedure compute-NES

```
def compute-NES($o$,$s$):
    select $v \in prevars(o)$ with $s(v) \neq pre(o)(v)$
    $N \leftarrow \{o' \in Act(s) \mid eff(o')(v) = pre(o)(v)\}$
    return $N$
```

Selection of $v \in prevars(o)$ arbitrary. Any variable will do.
Selection heuristics?

Algorithms

compute-interfering-operators: Compute interfering operators.

Precedure compute-interfering-operators (for strong SS)

```
def compute-interfering-operators($o$):
    disablers $\leftarrow \{o' \in O \mid o'$ disables $o\}$
    disablees $\leftarrow \{o' \in O \mid o$ disables $o'\}$
    conflicting $\leftarrow \{o' \in O \mid o$ and $o'$ conflict$\}$
    return disablers $\cup$ disablees $\cup$ conflicting
```

Precedure compute-interfering-operators (for weak SS)

```
def compute-interfering-operators($o$):
    disablees $\leftarrow \{o' \in O \mid o$ disables $o'\}$
    conflicting $\leftarrow \{o' \in O \mid o$ and $o'$ conflict$\}$
    return disablees $\cup$ conflicting
```

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Computing (strong and weak) stubborn sets for planning can be achieved with a fixpoint iteration until the constraints of T_s are satisfied:

compute-stubborn-set: Compute (strong or weak) stubborn set.

Precedure compute-stubborn-set

```
def compute-stubborn-set($s$):
    $T_s \leftarrow$ compute-DAL($\gamma$)
    while no fixed-point of $T_s$ reached do
        for $o \in T_s$ applicable in $s$:
            $T_s \leftarrow T_s \cup$ compute-interfering-operators($o$)
        for $o \in T_s$ not applicable in $s$:
            $T_s \leftarrow T_s \cup$ compute-NES($o$, $s$)
    end while
    return $T_s$
```

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Integration into A*

Observation: stubborn sets are state-dependent, but not path-dependent.

This allows filtering the applicable operators in s in graph search algorithms like A* that perform duplicate detection, too.

Instead of applying all applicable operators $app(s)$ in s, only apply operators in $T_{app(s)} := T_s \cap app(s)$.

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Preservation of Completeness and Optimality

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Theorem

Weak stubborn sets are completeness and optimality preserving.

Proof

Let $T_{app(s)} := T_s \cap app(s)$ for a weak stubborn set T_s.

We show that for all states s from which an optimal plan consisting of $n > 0$ operators exists, $T_{app(s)}$ contains an operator that starts such a plan.

We show by induction that A* restricting successor generation to $T_{app(s)}$ is optimal.

Let T_s be a weak stubborn set and $\pi = o_1, \ldots, o_n$ be an optimal plan that starts in s.

...

Preservation of Completeness and Optimality

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Proof (ctd.)

As T_s contains a disjunctive action landmark, π must contain an operator from T_s.

Let o_k be the operator with smallest index in π that is also contained in T_s, i.e., $o_k \in T_s$ and $\{o_1, \dots, o_{k-1}\} \cap T_s = \emptyset$.

We observe:

1. $o_k \in app(s)$: otherwise by definition of weak stubborn sets, a necessary enabling set N for o_k in s would have to be contained in T_s, and at least one operator from N would have to occur before o_k in π to enable o_k, contradicting that o_k was chosen with smallest index.
2. …

Preservation of Completeness and Optimality

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Proof (ctd.)

1. …
2. o_k is does not disable any of the operators $o_1, \dots, o_{k-1}$, and all these operators have non-conflicting effects with o_k: otherwise, as $o_k \in app(s)$, and by definition of weak stubborn sets, at least one of $o_1, \dots, o_{k-1}$ would have to be contained in T_s, again contradicting the assumption.

Hence, we can move o_k to the front:
$o_k, o_1, \dots, o_{k-1}, o_{k+1}, \dots, o_n$ is also a plan for Π.

It has the same cost as π and is hence optimal.

Thus, we have found an optimal plan of length n started by an operator $o_k \in T_{app(s)}$, completing the proof. □

Preservation of Completeness and Optimality

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Remark: The argument to move o_k to the front also holds for strong stubborn sets: in this case, o_k is not even disabled by any of $o_1, \ldots, o_{k-1}$ (and hence, o_k is independent of $o_1, \ldots, o_{k-1}$), which is a stronger property than needed in the proof.

Corollary

Strong stubborn sets are completeness and optimality preserving. □

Some Experiments: Overview

Optimal Planning, A* with LM-cut Heuristic, Selected Domains

	Coverage		Nodes generated	
Domain (problems)	A^*	+SSS	A^*	+SSS
PARCPRINTER-08 (30)	18	**+12**	2455181	$<$**1%**
PARCPRINTER-OPT11 (20)	13	**+7**	2454533	$<$**1%**
WOODWORKING-OPT08 (30)	17	**+10**	26796212	$<$**1%**
WOODWORKING-OPT11 (20)	12	**+7**	26795517	$<$**1%**
SATELLITE (36)	7	**+5**	5116312	**2%**
ROVERS (40)	7	**+2**	1900691	**22%**
AIRPORT (50)	**28**	$\pm$**0**	545072	**93%**
OPENSTACKS-OPT08 (30)	19	**+2**	56584063	**51%**
OPENSTACKS-OPT11 (20)	14	**+2**	56456969	**51%**
DRIVERLOG (20)	13	**+1**	3679376	**82%**
SCANALYZER-08 (30)	**15**	-3	**14203012**	**100%**
SCANALYZER-OPT11 (20)	**12**	-3	**14202884**	**100%**
PARKING-OPT11 (20)	**3**	-1	**560914**	**100%**
SOKOBAN-OPT08 (30)	**30**	-1	**20519270**	**100%**
VISITALL-OPT11 (20)	**11**	-1	**1991169**	**100%**
REMAINING DOMAINS (980)	**544**	$\pm$**0**	436017004	**93%**
SUM (1396)	763	**+39**	670278179	**77%**

Motivation
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Strong Stubborn Sets
Active Operators
Weak Stubborn Sets
Algorithms
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Some Experiments
Conclusion

Some Experiments

Weak compared to strong stubborn sets

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	Coverage		Nodes generated		# problems
Domain (problems)	SSS	WSS	SSS	WSS	w. diff. gen.
OPENSTACKS-OPT08 (30)	**21**	$\pm$**0**	152711917	**99.936%**	18
OPENSTACKS-OPT11 (20)	**16**	$\pm$**0**	152642101	**99.936%**	16
PATHWAYS-NONEG (30)	**5**	$\pm$**0**	162347	**99.702%**	2
PSR-SMALL (50)	**49**	$\pm$**0**	18119489	**99.998%**	6
SATELLITE (36)	**12**	$\pm$**0**	70299721	**92.804%**	12

$\Rightarrow$ In practice (or, at least, in the standard benchmark problems) there is no significant difference between weak and strong stubborn sets.

Motivation

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Stubborn Sets

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Conclusion

- Need for techniques orthogonal to heuristic search, complementing heuristics.
- One idea: Commit to one order of operators if they are independent. Prune other orders.
- Class of such techniques: partial-order reduction (POR)
- One such technique: strong/weak stubborn sets
- Can lead to substantial pruning compared to plain A*.
- Many other POR techniques exist.
- Other pruning techniques exist as well, e.g., symmetry reduction.