

Principles of AI Planning

5. Planning as search: progression and regression

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Planning as (classical) search

Search

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Summary

What do we mean by search?

- **Search** is a very generic term.
- ⇒ Every algorithm that tries out various alternatives can be said to “search” in some way.
- Here, we mean **classical search** algorithms.
 - **Search nodes** are **expanded** to generate **successor nodes**.
 - **Examples:** breadth-first search, A^* , hill-climbing, ...
- To be brief, we just say **search** in the following (not “classical search”).

Do you know this stuff already?

- We **assume prior knowledge** of basic search algorithms:
 - uninformed vs. informed
 - systematic vs. local
- There will be a small refresher in the next chapter.
- **Background:** Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)

- **search:** one of the **big success stories** of AI
- many planning algorithms based on classical AI search (we'll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)

Satisficing or optimal planning?

Must carefully distinguish two different problems:

- **satisficing planning**: any solution is OK
(although shorter solutions typically preferred)
- **optimal planning**: plans must have shortest possible length

Both are often solved by search, but:

- details are **very different**
- almost **no overlap** between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners

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How to apply search to planning? \rightsquigarrow many choices to make!

Choice 1: Search direction

- **progression**: forward from initial state to goal
- **regression**: backward from goal states to initial state
- **bidirectional search**

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Choice 2: Search space representation

- search nodes are associated with **states**
(\rightsquigarrow **state-space search**)
- search nodes are associated with **sets of states**

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How to apply search to planning? \rightsquigarrow many choices to make!

Choice 3: Search algorithm

- **uninformed search:**
depth-first, breadth-first, iterative depth-first, ...
- **heuristic search (systematic):**
greedy best-first, A^* , Weighted A^* , IDA^* , ...
- **heuristic search (local):**
hill-climbing, simulated annealing, beam search, ...

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How to apply search to planning? \rightsquigarrow **many choices to make!**

Choice 4: Search control

- **heuristics** for informed search algorithms
- **pruning techniques**: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, ...

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FF (Hoffmann & Nebel, 2001)

- **search direction:** forward search
- **search space representation:** single states
- **search algorithm:** enforced hill-climbing (informed local)
- **heuristic:** FF heuristic (inadmissible)
- **pruning technique:** helpful actions (incomplete)

⇒ one of the best satisficing planners

Fast Downward Stone Soup (Helmert et al., 2011)

- **search direction:** forward search
- **search space representation:** single states
- **search algorithm:** A^* (informed systematic)
- **heuristic:** multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, ...)
- **pruning technique:** none

⇒ one of the best optimal planners

Our plan for the next lectures

Choices to make:

- 1 search direction: progression/regression/both
~~~> **this chapter**
- 2 search space representation: states/sets of states  
~~~> **this chapter**
- 3 search algorithm: uninformed/heuristic; systematic/local
~~~> **next chapter**
- 4 search control: heuristics, pruning techniques  
~~~> **following chapters**

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Progression

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Progression: Computing the successor state $app_o(s)$ of a state s with respect to an operator o .

Progression planners find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and **progress it** through an operator, generating a new state
- solution found when a goal state generated

pro: very easy and efficient to implement

Two alternative search spaces for progression planners:

1 search nodes correspond to states

- when the same state is generated along different paths, it is not considered again (**duplicate detection**)
- **pro**: save time to consider same state again
- **con**: memory intensive (must maintain **closed list**)

2 search nodes correspond to operator sequences

- different operator sequences may lead to identical states (**transpositions**); search does not notice this
- **pro**: can be very memory-efficient
- **con**: much wasted work (often exponentially slower)

⇒ first alternative usually preferable in planning
(**unlike** many classical search benchmarks like 15-puzzle)

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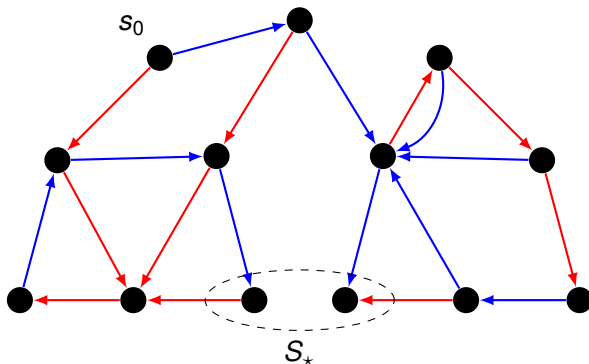
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Progression planning example (depth-first search)



Example where search nodes correspond to operator sequences
(no duplicate detection)



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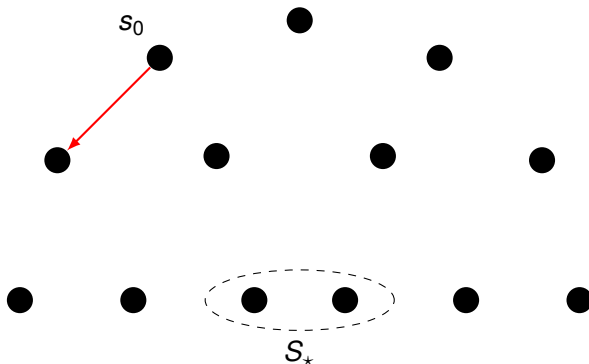
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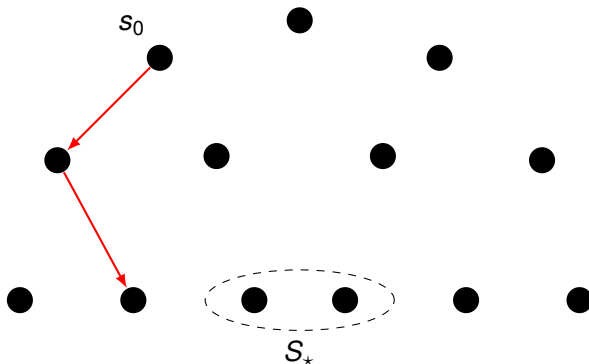
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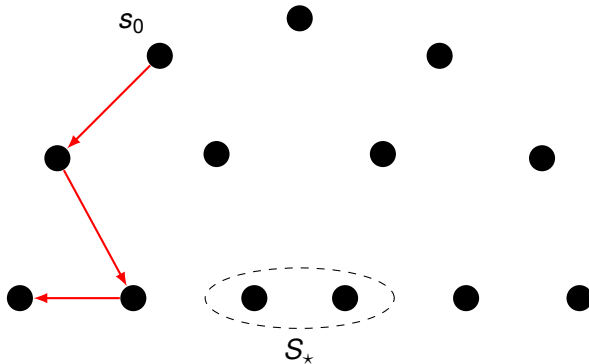
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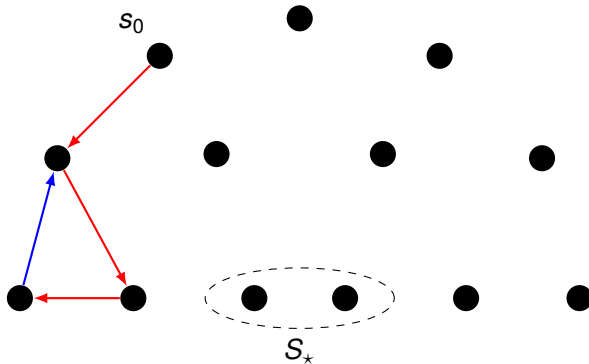
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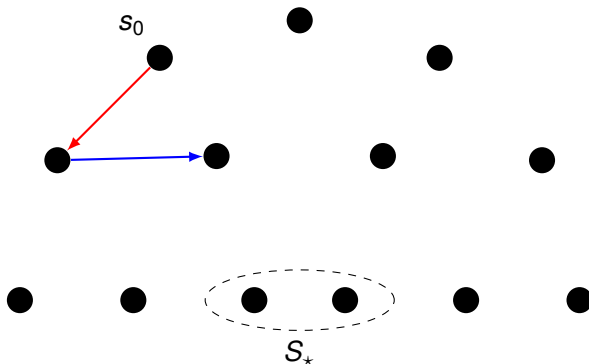
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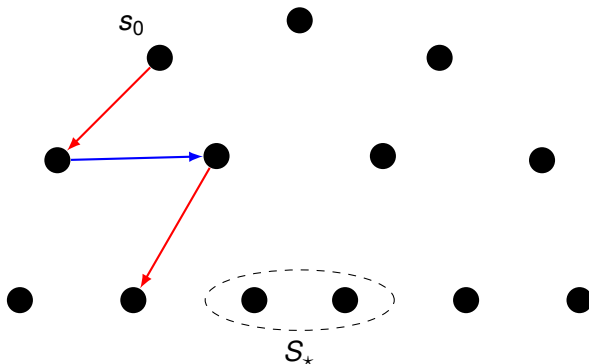
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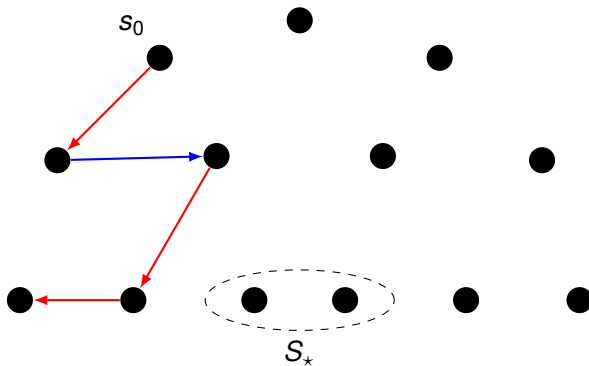
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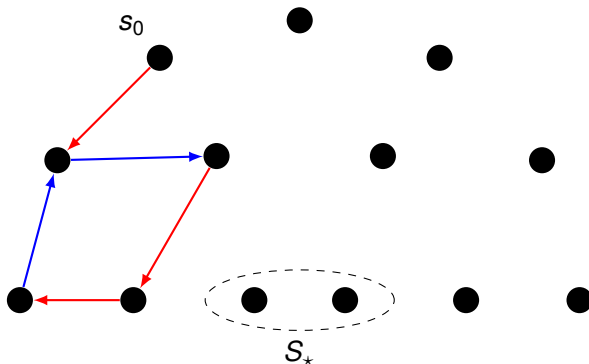
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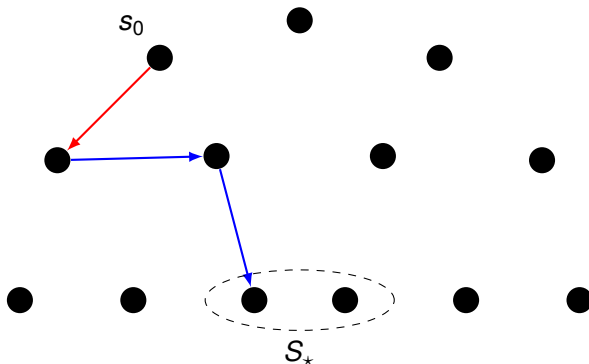
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Going through a transition graph in forward and backward directions is **not symmetric**:

- forward search starts from a **single** initial state;
backward search starts from a **set** of goal states
- when applying an operator o in a state s in forward direction, there is a **unique successor state** s' ;
if we applied operator o to end up in state s' ,
there can be **several possible predecessor states** s

~> most natural representation for backward search in planning associates **sets of states** with search nodes

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Regression: Computing the possible predecessor states $regr_o(G)$ of a set of states G with respect to the last operator o that was applied.

Regression planners find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated state set and **regress it** through an operator, generating a new state set
- solution found when a generated state set includes the initial state

Pro: can handle many states simultaneously

Con: basic operations complicated and expensive

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identify state sets with **logical formulae** (again):

- **search nodes correspond to state sets**
- each state set is represented by a **logical formula**:
 φ represents $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-hard or coNP-hard

Regression planning example (depth-first search)



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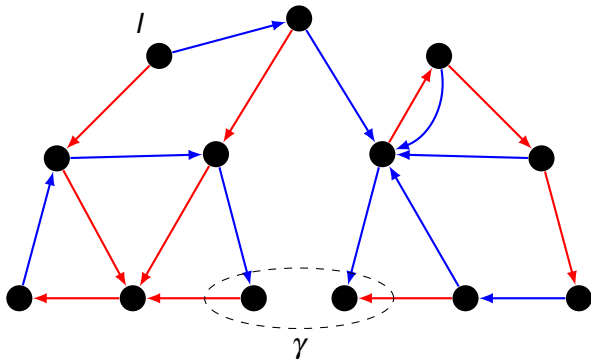
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Regression planning example (depth-first search)



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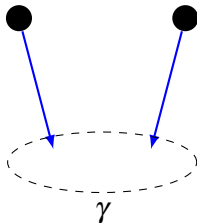


Regression planning example (depth-first search)



$$\varphi_1 = \text{regr}_{\rightarrow}(\gamma)$$

$$\varphi_1 \longrightarrow \gamma$$



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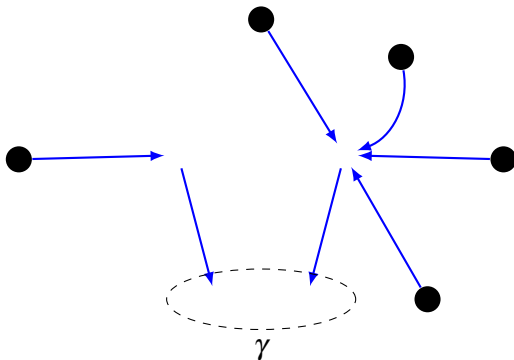
Regression planning example (depth-first search)



$$\varphi_1 = \text{regr} \rightarrow (\gamma)$$

$$\varphi_2 = \text{regr} \rightarrow (\varphi_1)$$

$$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$$



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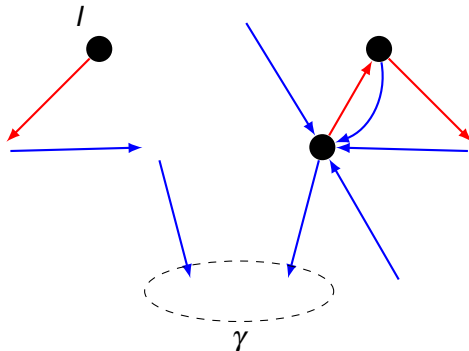


$$\varphi_1 = \text{regr}_{\rightarrow}(\gamma)$$

$$\varphi_2 = \text{regr}_{\rightarrow}(\varphi_1)$$

$$\varphi_3 = \text{regr}_{\rightarrow}(\varphi_2), I \models \varphi_3$$

$$\varphi_3 \xrightarrow{\text{red}} \varphi_2 \xrightarrow{\text{blue}} \varphi_1 \xrightarrow{\text{blue}} \gamma$$



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Definition (STRIPS planning task)

A planning task is a **STRIPS planning task** if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression **for STRIPS planning tasks** is very simple:

- Goals are conjunctions of atoms $a_1 \wedge \dots \wedge a_n$.
- **First step**: Choose an operator that makes none of a_1, \dots, a_n false.
- **Second step**: Remove goal atoms achieved by the operator (if any) and add its preconditions.

↪ Outcome of regression is again conjunction of atoms.

Optimization: only consider operators making some a_i true

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Definition (STRIPS regression)

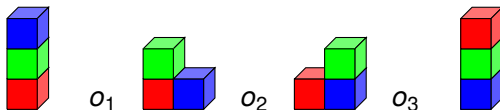
Let $\varphi = \varphi_1 \wedge \dots \wedge \varphi_n$ be a conjunction of atoms, and let $o = \langle \chi, e \rangle$ be a STRIPS operator which adds the atoms a_1, \dots, a_k and deletes the atoms d_1, \dots, d_l .

The **STRIPS regression** of φ with respect to o is

$$\text{sregr}_o(\varphi) := \begin{cases} \perp & \text{if } a_i = d_j \text{ for some } i, j \\ \perp & \text{if } \varphi_i = d_j \text{ for some } i, j \\ \chi \wedge \wedge(\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{otherwise} \end{cases}$$

Note: $\text{sregr}_o(\varphi)$ is again a conjunction of atoms, or \perp .

STRIPS regression example



Note: Predecessor states are in general not unique.
This picture is just for illustration purposes.

$$\begin{aligned}
 o_1 &= \langle \text{blue on green} \wedge \text{blue clr}, & \neg \text{blue on green} \wedge \text{blue onT} \wedge \text{green clr} \rangle \\
 o_2 &= \langle \text{green on red} \wedge \text{green clr} \wedge \text{blue clr}, & \neg \text{blue clr} \wedge \neg \text{green on red} \wedge \text{green on blue} \wedge \text{red clr} \rangle \\
 o_3 &= \langle \text{red onT} \wedge \text{red clr} \wedge \text{green clr}, & \neg \text{green clr} \wedge \neg \text{red onT} \wedge \text{red on green} \rangle
 \end{aligned}$$

$$\gamma = \text{red on green} \wedge \text{green on blue}$$

$$\phi_1 = \text{sregr}_{o_3}(\gamma) = \text{red onT} \wedge \text{red clr} \wedge \text{green clr} \wedge \text{green on blue}$$

$$\phi_2 = \text{sregr}_{o_2}(\phi_1) = \text{green on red} \wedge \text{green clr} \wedge \text{blue clr} \wedge \text{red onT}$$

$$\phi_3 = \text{sregr}_{o_1}(\phi_2) = \text{blue on green} \wedge \text{blue clr} \wedge \text{green on red} \wedge \text{red onT}$$

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- With disjunctions and conditional effects, things become more tricky. How to regress $a \vee (b \wedge c)$ with respect to $\langle q, d \triangleright b \rangle$?
- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.
- We present a general method for doing regression for any formula and any operator.
- Now we extensively use the idea of representing sets of states as formulae.

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Definition (effect precondition)

The **effect precondition** $EPC_I(e)$ for literal I and effect e is defined as follows:

$$\begin{aligned}EPC_I(I) &= \top \\EPC_I(I') &= \perp \text{ if } I \neq I' \quad (\text{for literals } I') \\EPC_I(e_1 \wedge \dots \wedge e_n) &= EPC_I(e_1) \vee \dots \vee EPC_I(e_n) \\EPC_I(\chi \triangleright e) &= EPC_I(e) \wedge \chi\end{aligned}$$

Intuition: $EPC_I(e)$ describes the situations in which effect e causes literal I to become true.

Example

$$EPC_a(b \wedge c) = \perp \vee \perp \equiv \perp$$

$$EPC_a(a \wedge (b \triangleright a)) = \top \vee (\top \wedge b) \equiv \top$$

$$EPC_a((c \triangleright a) \wedge (b \triangleright a)) = (\top \wedge c) \vee (\top \wedge b) \equiv c \vee b$$

Lemma (A)

*Let s be a state, l a literal and e an effect.
Then $l \in [e]_s$ if and only if $s \models EPC_l(e)$.*

Proof.

Induction on the structure of the effect e .

Base case 1, $e = l: l \in [l]_s = \{l\}$ by definition, and $s \models EPC_l(l) = \top$ by definition. Both sides of the equivalence are true.

Base case 2, $e = l'$ for some literal $l' \neq l: l \notin [l']_s = \{l'\}$ by definition, and $s \not\models EPC_l(l') = \perp$ by definition. Both sides are false.

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Proof (ctd.)

Inductive case 1, $e = e_1 \wedge \dots \wedge e_n$:

$I \in [e]_s$ iff $I \in [e_1]_s \cup \dots \cup [e_n]_s$ (Def $[e_1 \wedge \dots \wedge e_n]_s$)

iff $I \in [e']_s$ for some $e' \in \{e_1, \dots, e_n\}$

iff $s \models EPC_I(e')$ for some $e' \in \{e_1, \dots, e_n\}$ (IH)

iff $s \models EPC_I(e_1) \vee \dots \vee EPC_I(e_n)$

iff $s \models EPC_I(e_1 \wedge \dots \wedge e_n)$. (Def EPC)

Inductive case 2, $e = \chi \triangleright e'$:

$I \in [\chi \triangleright e']_s$ iff $I \in [e']_s$ and $s \models \chi$ (Def $[\chi \triangleright e']_s$)

iff $s \models EPC_I(e')$ and $s \models \chi$ (IH)

iff $s \models EPC_I(e') \wedge \chi$

iff $s \models EPC_I(\chi \triangleright e')$. (Def EPC)



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Remark: *EPC* vs. effect normal form

Notice that in terms of $EPC_a(e)$, any operator $\langle \chi, e \rangle$ can be expressed in effect normal form as

$$\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \triangleright a) \wedge (EPC_{\neg a}(e) \triangleright \neg a)) \right\rangle,$$

where A is the set of all state variables.

The formula $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$ expresses the **value of state variable $a \in A$ after applying o** in terms of **values of state variables before applying o** .

Either:

- a became true, or
- a was true before and it did not become false.

Example

Let $e = (b \triangleright a) \wedge (c \triangleright \neg a) \wedge b \wedge \neg d$.

| variable x | $EPC_x(e) \vee (x \wedge \neg EPC_{\neg x}(e))$ |
|--------------|---|
| a | $b \vee (a \wedge \neg c)$ |
| b | $\top \vee (b \wedge \neg \perp) \equiv \top$ |
| c | $\perp \vee (c \wedge \neg \perp) \equiv c$ |
| d | $\perp \vee (d \wedge \neg \top) \equiv \perp$ |

Lemma (B)

Let a be a state variable, $o = \langle \chi, e \rangle$ an operator, s a state, and $s' = \text{app}_o(s)$.

Then $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$ if and only if $s' \models a$.

Proof.

(\Rightarrow) : Assume $s \models \text{EPC}_a(e) \vee (a \wedge \neg \text{EPC}_{\neg a}(e))$.

Do a case analysis on the two disjuncts.

- 1 Assume that $s \models \text{EPC}_a(e)$. By Lemma A, we have $a \in [e]_s$ and hence $s' \models a$.
- 2 Assume that $s \models a \wedge \neg \text{EPC}_{\neg a}(e)$. By Lemma A, we have $\neg a \notin [e]_s$. Hence a remains true in s' .

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Proof (ctd.)

(\Leftarrow): We showed that if the formula is **true** in s , then a is **true** in s' . For the second part, we show that if the formula is **false** in s , then a is **false** in s' .

- So assume $s \not\models EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.
 - Then $s \models \neg EPC_a(e) \wedge (\neg a \vee EPC_{\neg a}(e))$ (de Morgan).
 - Case distinction: a is true or a is false in s .
 - 1 Assume that $s \models a$. Now $s \models EPC_{\neg a}(e)$ because $s \models \neg a \vee EPC_{\neg a}(e)$.
Hence by Lemma A $\neg a \in [e]_s$ and we get $s' \not\models a$.
 - 2 Assume that $s \not\models a$. Because $s \models \neg EPC_a(e)$, by Lemma A we get $a \notin [e]_s$ and hence $s' \not\models a$.
- Therefore in both cases $s' \not\models a$.

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We base the definition of regression on formulae $EPC_I(e)$.

Definition (general regression)

Let φ be a propositional formula and $o = \langle \chi, e \rangle$ an operator.
The **regression of φ with respect to o** is

$$\text{regr}_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$$

where

- 1 φ_r is obtained from φ by replacing each $a \in A$ by $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$, and
- 2 $\kappa = \bigwedge_{a \in A} \neg(EPC_a(e) \wedge EPC_{\neg a}(e))$.

The formula κ expresses that operators are only applicable in states where their change sets are consistent.

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$$\blacksquare \text{regr}_{\langle a, b \rangle}(b) \equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge \top \equiv a$$

$$\begin{aligned} \blacksquare \text{regr}_{\langle a, b \rangle}(b \wedge c \wedge d) \\ &\equiv a \wedge (\top \vee (b \wedge \neg \perp)) \wedge (\perp \vee (c \wedge \neg \perp)) \wedge (\perp \vee (d \wedge \neg \perp)) \wedge \top \\ &\equiv a \wedge c \wedge d \end{aligned}$$

$$\blacksquare \text{regr}_{\langle a, c \triangleright b \rangle}(b) \equiv a \wedge (c \vee (b \wedge \neg \perp)) \wedge \top \equiv a \wedge (c \vee b)$$

$$\begin{aligned} \blacksquare \text{regr}_{\langle a, (c \triangleright b) \wedge (b \triangleright \neg b) \rangle}(b) &\equiv a \wedge (c \vee (b \wedge \neg b)) \wedge \neg(c \wedge b) \\ &\equiv a \wedge c \wedge \neg b \end{aligned}$$

$$\begin{aligned} \blacksquare \text{regr}_{\langle a, (c \triangleright b) \wedge (d \triangleright \neg b) \rangle}(b) &\equiv a \wedge (c \vee (b \wedge \neg d)) \wedge \neg(c \wedge d) \\ &\equiv a \wedge (c \vee b) \wedge (c \vee \neg d) \wedge (\neg c \vee \neg d) \\ &\equiv a \wedge (c \vee b) \wedge \neg d \end{aligned}$$

Regression example: binary counter



$$\begin{aligned} & (\neg b_0 \triangleright b_0) \wedge \\ & ((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ & ((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

$$EPC_{b_2}(e) = \neg b_2 \wedge b_1 \wedge b_0$$

$$EPC_{b_1}(e) = \neg b_1 \wedge b_0$$

$$EPC_{b_0}(e) = \neg b_0$$

$$EPC_{\neg b_2}(e) = \perp$$

$$EPC_{\neg b_1}(e) = \neg b_2 \wedge b_1 \wedge b_0$$

$$EPC_{\neg b_0}(e) = (\neg b_1 \wedge b_0) \vee (\neg b_2 \wedge b_1 \wedge b_0) \equiv (\neg b_1 \vee \neg b_2) \wedge b_0$$

Regression replaces state variables as follows:

$$b_2 \quad \text{by} \quad (\neg b_2 \wedge b_1 \wedge b_0) \vee (b_2 \wedge \neg \perp) \equiv (b_1 \wedge b_0) \vee b_2$$

$$\begin{aligned} b_1 \quad \text{by} \quad & (\neg b_1 \wedge b_0) \vee (b_1 \wedge \neg(\neg b_2 \wedge b_1 \wedge b_0)) \\ & \equiv (\neg b_1 \wedge b_0) \vee (b_1 \wedge (b_2 \vee \neg b_0)) \end{aligned}$$

$$b_0 \quad \text{by} \quad \neg b_0 \vee (b_0 \wedge \neg((\neg b_1 \vee \neg b_2) \wedge b_0)) \equiv \neg b_0 \vee (b_1 \wedge b_2)$$

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Theorem (correctness of $\text{regr}_o(\varphi)$)

Let φ be a formula, o an operator and s a state.

Then $s \models \text{regr}_o(\varphi)$ iff o is applicable in s and $\text{app}_o(s) \models \varphi$.

Proof.

Let $o = \langle \chi, e \rangle$. Recall that $\text{regr}_o(\varphi) = \chi \wedge \varphi_r \wedge \kappa$, where φ_r and κ are as defined previously.

If o is inapplicable in s , then $s \not\models \chi \wedge \kappa$, both sides of the “iff” condition are false, and we are done. Hence, we only further consider states s where o is applicable. Let $s' := \text{app}_o(s)$.

We know that $s \models \chi \wedge \kappa$ (because o is applicable), so the “iff” condition we need to prove simplifies to:

$$s \models \varphi_r \text{ iff } s' \models \varphi.$$

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Proof (ctd.)

To show: $s \models \varphi_r$ iff $s' \models \varphi$.

We show that for all formulae ψ , $s \models \psi_r$ iff $s' \models \psi$, where ψ_r is ψ with every $a \in A$ replaced by $EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.

The proof is by structural induction on ψ .

Induction hypothesis $s \models \psi_r$ if and only if $s' \models \psi$.

Base cases 1 & 2 $\psi = \top$ or $\psi = \perp$: trivial, as $\psi_r = \psi$.

Base case 3 $\psi = a$ for some $a \in A$:

Then $\psi_r = EPC_a(e) \vee (a \wedge \neg EPC_{\neg a}(e))$.

By Lemma B, $s \models \psi_r$ iff $s' \models \psi$.

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Proof (ctd.)

Inductive case 1 $\psi = \neg\psi'$:

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\neg\psi')_r \text{ iff } s \models \neg(\psi'_r) \text{ iff } s \not\models \psi'_r \\ \text{iff (IH) } s' \not\models \psi' \text{ iff } s' \models \neg\psi' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 2 $\psi = \psi' \vee \psi''$:

$$\begin{aligned} s \models \psi_r \text{ iff } s \models (\psi' \vee \psi'')_r \text{ iff } s \models \psi'_r \vee \psi''_r \\ \text{iff } s \models \psi'_r \text{ or } s \models \psi''_r \\ \text{iff (IH, twice) } s' \models \psi' \text{ or } s' \models \psi'' \\ \text{iff } s' \models \psi' \vee \psi'' \text{ iff } s' \models \psi \end{aligned}$$

Inductive case 3 $\psi = \psi' \wedge \psi''$: Very similar to inductive case 2, just with \wedge instead of \vee and “and” instead of “or”.

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The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $regr_o(\varphi)$ does not represent the empty set (which would mean that search is in a dead end).
For example, $regr_{\langle a, \neg p \rangle}(p) \equiv a \wedge \perp \equiv \perp$.
- Test that $regr_o(\varphi)$ does not represent a subset of φ (which would make the problem harder than before).
For example, $regr_{\langle b, c \rangle}(a) \equiv a \wedge b$.

Both of these problems are **NP-hard**.

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The formula $\text{regr}_{o_1}(\text{regr}_{o_2}(\dots \text{regr}_{o_{n-1}}(\text{regr}_{o_n}(\varphi))))$ may have size $O(|\varphi||o_1||o_2|\dots|o_{n-1}||o_n|)$, i. e., the product of the sizes of φ and the operators.

\rightsquigarrow worst-case **exponential** size $O(m^n)$

Logical simplifications

- $\perp \wedge \varphi \equiv \perp, \top \wedge \varphi \equiv \varphi, \perp \vee \varphi \equiv \varphi, \top \vee \varphi \equiv \top$
- $a \vee \varphi \equiv a \vee \varphi[\perp/a], \neg a \vee \varphi \equiv \neg a \vee \varphi[\top/a],$
 $a \wedge \varphi \equiv a \wedge \varphi[\top/a], \neg a \wedge \varphi \equiv \neg a \wedge \varphi[\perp/a]$
- idempotency, absorption, commutativity, associativity, ...

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Problem very big formulae obtained by regression

Cause **disjunctivity** in the (NNF) formulae
(formulae **without disjunctions** easily convertible to
small formulae $l_1 \wedge \dots \wedge l_n$ where l_i are literals and n
is at most the number of state variables.)

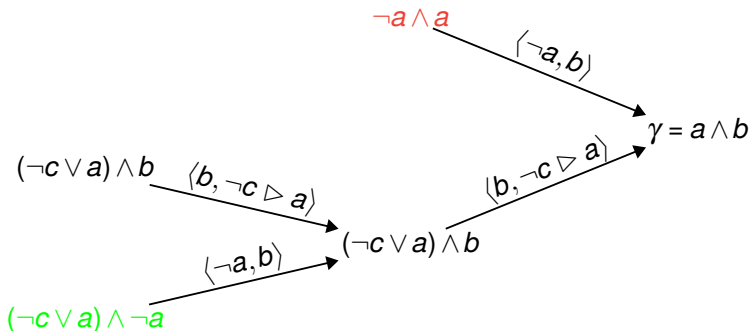
Idea handle disjunctivity when generating search trees

Unrestricted regression: search tree example



Unrestricted regression: do not treat disjunctions specially

Goal $\gamma = a \wedge b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.



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Alternatives:

- 1 Do nothing (**unrestricted regression**).
- 2 Always eliminate all disjunctivity (**full splitting**).
- 3 Reduce disjunctivity if formula becomes too big.

Discussion:

- With **unrestricted regression** the formulae may have **size that is exponential** in the number of state variables.
- With **full splitting** search tree can be **exponentially bigger** than without splitting.
- The third option lies between these two extremes.

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Summary

- (Classical) **search** is a very important planning approach.
- Search-based planning algorithms differ along many dimensions, including
 - **search direction** (forward, backward)
 - **what each search node represents**
(a state, a set of states, an operator sequence)
- **Progression search** proceeds forwards from the initial state.
 - If we use duplicate detection, each search node corresponds to a unique **state**.
 - If we do not use duplicate detection, each search node corresponds to a unique **operator sequence**.

- **Regression search** proceeds backwards from the goal.
 - Each search node corresponds to a **set of states** represented by a **formula**.
 - Regression is simple for **STRIPS** operators.
 - The theory for **general regression** is more complex.
 - When applying regression in practice, additional considerations such as when and how to perform **splitting** come into play.