

Principles of AI Planning

3. Normal forms

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Similarly to normal forms in propositional logic (DNF, CNF, NNF, ...) we can define **normal forms for effects, operators and planning tasks**.

This is useful because algorithms (and proofs) then only need to deal with effects (resp. operators or tasks) in normal form.



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Definition (equivalent effects)

Two effects e and e' over state variables A are **equivalent**, written $e \equiv e'$, if for all states s over A , $[e]_s = [e']_s$.

Definition (equivalent operators)

Two operators o and o' over state variables A are **equivalent**, written $o \equiv o'$, if they are applicable in the same states, and for all states s where they are applicable, $app_o(s) = app_{o'}(s)$.

Theorem

Let $o = \langle \chi, e \rangle$ and $o' = \langle \chi', e' \rangle$ be operators with $\chi \equiv \chi'$ and $e \equiv e'$. Then $o \equiv o'$.

Note: The converse is not true. (Why not?)

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$$e_1 \wedge e_2 \equiv e_2 \wedge e_1 \quad (1)$$

$$(e_1 \wedge e_2) \wedge e_3 \equiv e_1 \wedge (e_2 \wedge e_3) \quad (2)$$

$$\top \wedge e \equiv e \quad (3)$$

$$\chi \triangleright e \equiv \chi' \triangleright e \quad \text{if } \chi \equiv \chi' \quad (4)$$

$$\top \triangleright e \equiv e \quad (5)$$

$$\perp \triangleright e \equiv \top \quad (6)$$

$$\chi_1 \triangleright (\chi_2 \triangleright e) \equiv (\chi_1 \wedge \chi_2) \triangleright e \quad (7)$$

$$\chi \triangleright (e_1 \wedge \cdots \wedge e_n) \equiv (\chi \triangleright e_1) \wedge \cdots \wedge (\chi \triangleright e_n) \quad (8)$$

$$(\chi_1 \triangleright e) \wedge (\chi_2 \triangleright e) \equiv (\chi_1 \vee \chi_2) \triangleright e \quad (9)$$

We can define a **normal form for effects**:

- Nesting of conditionals, as in $a \triangleright (b \triangleright c)$, can be eliminated.
- Effects e within a conditional effect $\varphi \triangleright e$ can be restricted to atomic effects (a or $\neg a$).

Transformation to this effect normal form only gives a small polynomial size increase.

Compare: transformation to CNF or DNF may increase formula size exponentially.

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An operator $\langle \chi, e \rangle$ is in **effect normal form (ENF)** if for all occurrences of $\chi' \triangleright e'$ in e the effect e' is either a or $\neg a$ for some $a \in A$, and there is at most one occurrence of any atomic effect in e .

Theorem

For every operator there is an equivalent one in effect normal form.

Proof is constructive: we can transform any operator into effect normal form using the equivalence transformations for effects.

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$$\begin{aligned} &(a \triangleright (b \wedge \\ &\quad (c \triangleright (\neg d \wedge e)))) \wedge \\ &(\neg b \triangleright e) \end{aligned}$$

transformed to effect normal form is

$$\begin{aligned} &(a \triangleright b) \wedge \\ &((a \wedge c) \triangleright \neg d) \wedge \\ &((\neg b \vee (a \wedge c)) \triangleright e) \end{aligned}$$

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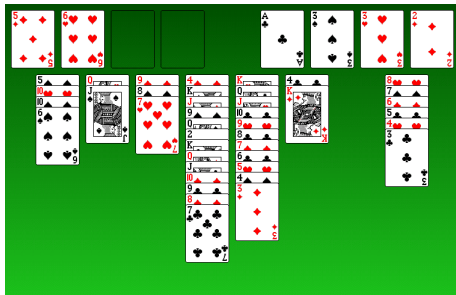
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Example: Freecell



Example (good and bad effects)

If we move a card c to a free tableau position, the **good effect** is that the card formerly below c is now available.

The **bad effect** is that we lose one free tableau position.

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What is a good or bad effect?

Question: Which operator effects are good, and which are bad?

Difficult to answer in general, because it depends on context:

- Locking the entrance door is **good** if we want to keep burglars out.
- Locking the entrance door is **bad** if we want to enter.

We will now consider a reformulation of planning tasks that makes the distinction between good and bad effects obvious.

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Definition (operators in positive normal form)

An operator $o = \langle \chi, e \rangle$ is in **positive normal form** if it is in effect normal form, no negation symbols appear in χ , and no negation symbols appear in any effect condition in e .

Definition (planning tasks in positive normal form)

A planning task $\langle A, I, O, \gamma \rangle$ is in **positive normal form** if all operators in O are in positive normal form and no negation symbols occur in the goal γ .

Theorem (positive normal form)

Every planning task Π has an equivalent planning task Π' in positive normal form.

Moreover, Π' can be computed from Π in polynomial time.

Note: Equivalence here means that the represented transition systems of Π and Π' , limited to the states that can be reached from the initial state, are isomorphic.

We prove the theorem by describing a suitable algorithm.
(However, we do not prove its correctness or complexity.)

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Transformation of $\langle A, I, O, \gamma \rangle$ to positive normal form

Convert all operators $o \in O$ to effect normal form.

Convert all conditions to negation normal form (NNF).

while any condition contains a negative literal $\neg a$:

Let a be a variable which occurs negatively in a condition.

$A := A \cup \{\hat{a}\}$ for some new state variable \hat{a}

$I(\hat{a}) := 1 - I(a)$

Replace the effect a by $(a \wedge \neg \hat{a})$ in all operators $o \in O$.

Replace the effect $\neg a$ by $(\neg a \wedge \hat{a})$ in all operators $o \in O$.

Replace $\neg a$ by \hat{a} in all conditions.

Convert all operators $o \in O$ to effect normal form (again).

Here, *all conditions* refers to all operator preconditions, operator effect conditions and the goal.

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Example (transformation to positive normal form)

$A = \{home, uni, lecture, bike, bike-locked\}$

$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0\}$

$O = \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\ \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$

$\gamma = lecture \wedge bike$

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Identify state variable a occurring negatively in conditions.

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Example (transformation to positive normal form)

$$A = \{home, uni, lecture, bike, bike-locked, \textcolor{red}{bike-unlocked}\}$$

$$I = \{home \mapsto 1, bike \mapsto 1, bike-locked \mapsto 1, \\ uni \mapsto 0, lecture \mapsto 0, \textcolor{red}{bike-unlocked} \mapsto 0\}$$

$$O = \{\langle home \wedge bike \wedge \neg bike-locked, \neg home \wedge uni \rangle, \\ \langle bike \wedge bike-locked, \neg bike-locked \rangle, \\ \langle bike \wedge \neg bike-locked, bike-locked \rangle, \\ \langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$$

$$\gamma = lecture \wedge bike$$

Introduce new variable \hat{a} with complementary initial value.

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 $\langle bike \wedge \neg bike-locked, bike-locked \rangle,$
 $\langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$

$\gamma = lecture \wedge bike$

Identify effects on variable *a*.

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$A = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$

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 $\langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle,$
 $\langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle,$
 $\langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$

$\gamma = lecture \wedge bike$

Introduce complementary effects for \hat{a} .

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 $\langle bike \wedge \neg bike-locked, bike-locked \wedge \neg bike-unlocked \rangle,$
 $\langle uni, lecture \wedge ((bike \wedge \neg bike-locked) \triangleright \neg bike) \rangle\}$

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Identify negative conditions for a .

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$A = \{home, uni, lecture, bike, bike-locked, bike-unlocked\}$

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$O = \{ \langle home \wedge bike \wedge bike-unlocked, \neg home \wedge uni \rangle,$
 $\langle bike \wedge bike-locked, \neg bike-locked \wedge bike-unlocked \rangle,$
 $\langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle,$
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$\gamma = lecture \wedge bike$

Replace by positive condition \hat{a} .

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 $\langle bike \wedge bike-unlocked, bike-locked \wedge \neg bike-unlocked \rangle,$
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$\gamma = lecture \wedge bike$

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Why positive normal form is interesting

In positive normal form, good and bad effects are easy to distinguish:

- Effects that make state variables true are good (add effects).
- Effects that make state variables false are bad (delete effects).

This is of high relevance for some planning techniques that we will see later in this course.

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An operator $\langle \chi, e \rangle$ is a **STRIPS operator** if

- χ is a conjunction of atoms, and
- e is a conjunction of atomic effects.

Hence every STRIPS operator is of the form

$$\langle a_1 \wedge \dots \wedge a_n, l_1 \wedge \dots \wedge l_m \rangle$$

where a_i are atoms and l_j are atomic effects.

Note: Sometimes we allow conjunctions of **literals** as preconditions. We denote this as **STRIPS with negative preconditions**.

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Why STRIPS is interesting

- STRIPS operators are **particularly simple**, yet expressive enough to capture general planning problems.
- In particular, STRIPS planning is **no easier** than general planning problems.
- Many algorithms in the planning literature are **only presented for STRIPS operators** (generalization is often, but not always, obvious).

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STRIPS

Stanford Research Institute Planning System
(Fikes & Nilsson, 1971)

- Not every operator is equivalent to a STRIPS operator.
- However, each operator can be transformed into a **set** of STRIPS operators whose “combination” is equivalent to the original operator. (How?)
- However, this transformation may exponentially increase the number of required operators. There are planning tasks for which such a blow-up is unavoidable.
- There are polynomial transformations of planning tasks to STRIPS, but these do not preserve the structure of the transition system (e. g., length of shortest plans may change).

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- **Effect normal form** simplifies structure of operator effects: conditional effects contain only atomic effects; there is at most one occurrence of any atomic effect.
- **Positive normal form** allows to distinguish good and bad effects.
- The form of **STRIPS operators** is even more restrictive than effect normal form, forbidding complex preconditions and conditional effects.
- All three forms are expressive enough to capture general planning problems.
- Transformation to effect normal form and positive normal form possible with polynomial size increase.
- Structure preserving transformations of planning tasks to STRIPS can increase the number of operators exponentially.

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