

Principles of AI Planning

2. Transition systems and planning tasks

Albert-Ludwigs-Universität Freiburg



**UNI
FREIBURG**

Bernhard Nebel and Robert Mattmüller

October 16th, 2017



Transition systems

Definition (transition system)

A **transition system** is a 5-tuple $\mathcal{T} = \langle S, L, T, s_0, S_\star \rangle$ where

- S is a finite set of **states**,
- L is a finite set of (transition) **labels**,
- $T \subseteq S \times L \times S$ is the **transition relation**,
- $s_0 \in S$ is the **initial state**, and
- $S_\star \subseteq S$ is the set of **goal states**.

We say that \mathcal{T} **has the transition** $\langle s, \ell, s' \rangle$ if $\langle s, \ell, s' \rangle \in T$.

We also write this $s \xrightarrow{\ell} s'$, or $s \rightarrow s'$ when not interested in ℓ .

Note: Transition systems are also called **state spaces**.

Transition
systems

Definition

Blocks world

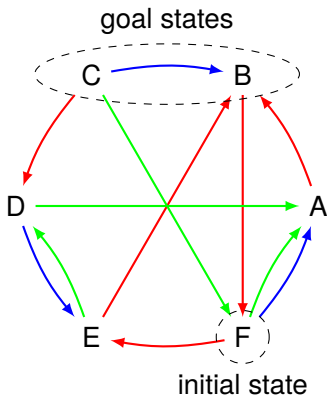
Planning
tasks

Summary

Transition systems: example



Transition systems are often depicted as **directed arc-labeled graphs** with marks to indicate the initial state and goal states.



Transition
systems

Definition

Blocks world

Planning
tasks

Summary

We use common graph theory terms for transition systems:

- s' **successor** of s if $s \rightarrow s'$
- s **predecessor** of s' if $s \rightarrow s'$
- s' **reachable** from s if there exists a sequence of transitions

$$s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n \text{ s.t. } s^0 = s \text{ and } s^n = s'$$

- **Note:** $n = 0$ possible; then $s = s'$
- $s^0 \xrightarrow{\ell_1} s^1, \dots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called **path** from s to s'
- s^0, \dots, s^n is also called **path** from s to s'
- **length** of that path is n
- additional terms: **strongly connected**, **weakly connected**, **strong/weak connected components**, ...

Transition
systems

Definition

Blocks world

Planning
tasks

Summary



Some additional terminology:

- s' **reachable** (without reference state) means reachable from initial state s_0
- **solution** or **goal path** from s : path from s to some $s' \in S_*$
 - if s is omitted, $s = s_0$ is implied
- transition system **solvable** if a goal path from s_0 exists

Definition (deterministic transition system)

A transition system with transitions T is called **deterministic** if for all states s and labels ℓ , there is **at most one** state s' with $s \xrightarrow{\ell} s'$.

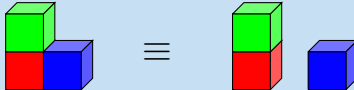
Example: previously shown transition system

Running example: blocks world



- Throughout the course, we will often use the **blocks world** domain as an example.
- In the blocks world, a number of differently coloured blocks are arranged on our table.
- Our job is to rearrange them according to a given goal.

Location on the table does not matter.



Location on a block does not matter.



Transition
systems

Definition

Blocks world

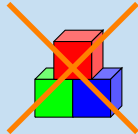
Planning
tasks

Summary

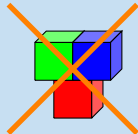
Blocks world rules (ctd.)



At most one block may be below a block.



At most one block may be on top of a block.



Transition
systems

Definition

Blocks world

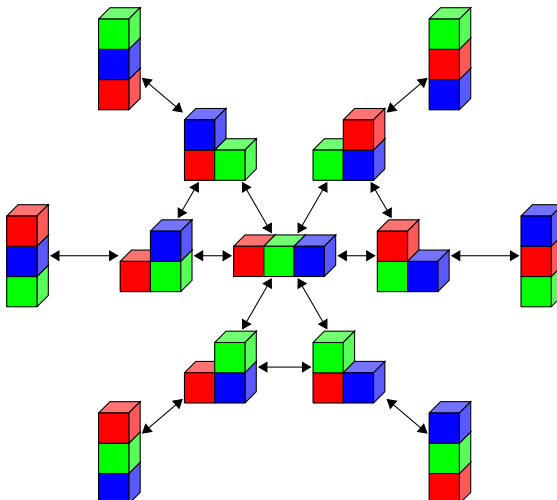
Planning
tasks

Summary

Blocks world transition system for three blocks



(Transition labels omitted for clarity.)



Transition
systems

Definition

Blocks world

Planning
tasks

Summary

blocks	states	blocks	states
1	1	10	58941091
2	3	11	824073141
3	13	12	12470162233
4	73	13	202976401213
5	501	14	3535017524403
6	4051	15	65573803186921
7	37633	16	1290434218669921
8	394353	17	26846616451246353
9	4596553	18	588633468315403843

- Finding a solution is polynomial time in the number of blocks (move everything onto the table and then construct the goal configuration).
- Finding a shortest solution is NP-complete (for a compact description of the problem).



Planning tasks

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary



- Classical (i. e., deterministic) planning is in essence the problem of finding solutions in **huge** transition systems.
- The transition systems we are usually interested in are too large to explicitly enumerate all states or transitions.
- Hence, the input to a planning algorithm must be given in a more **concise** form.
- In the rest of chapter, we discuss how to represent planning tasks in a suitable way.

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

How to represent huge state sets without enumerating them?

- represent different aspects of the world in terms of different **state variables**

~> a state is a **valuation of state variables**

- n state variables with m possible values each induce m^n different states

~> **exponentially more compact** than “flat” representations

- **Example:** n variables suffice for blocks world with n blocks

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Blocks world with finite-domain state variables



Describe blocks world state with three state variables:

- *location-of-A*: {B, C, table}
- *location-of-B*: {A, C, table}
- *location-of-C*: {A, B, table}

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

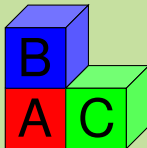
Summary

Example

$s(\text{location-of-A}) = \text{table}$

$s(\text{location-of-B}) = A$

$s(\text{location-of-C}) = \text{table}$



Not all valuations correspond to intended blocks world states.

Example: s with $s(\text{location-of-A}) = B$, $s(\text{location-of-B}) = A$.

Problem:

- How to **succinctly** represent **transitions** and **goal states**?

Idea: Use **propositional logic**

- **state variables**: propositional variables (0 or 1)
- **goal states**: defined by a propositional formula
- **transitions**: defined by **actions** given by
 - **precondition**: when is the action applicable?
 - **effect**: how does it change the valuation?

Note: general finite-domain state variables can be compactly encoded as Boolean variables

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

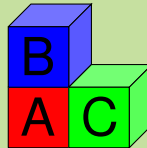
Summary

Blocks world with Boolean state variables



Example

$s(A\text{-on-}B) = 0$
 $s(A\text{-on-}C) = 0$
 $s(A\text{-on-table}) = 1$
 $s(B\text{-on-}A) = 1$
 $s(B\text{-on-}C) = 0$
 $s(B\text{-on-table}) = 0$
 $s(C\text{-on-}A) = 0$
 $s(C\text{-on-}B) = 0$
 $s(C\text{-on-table}) = 1$



Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (propositional formula)

Let A be a set of **atomic propositions** (here: state variables).

The **propositional formulae** over A are constructed by finite application of the following rules:

- \top and \perp are propositional formulae (**truth** and **falsity**).
- For all $a \in A$, a is a propositional formula (**atom**).
- If ϕ is a propositional formula, then so is $\neg\phi$ (**negation**).
- If ϕ and ψ are propositional formulas, then so are $(\phi \vee \psi)$ (**disjunction**) and $(\phi \wedge \psi)$ (**conjunction**).

Note: We often omit the word “propositional”.

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary



Abbreviations:

- $(\varphi \rightarrow \psi)$ is short for $(\neg\varphi \vee \psi)$ (**implication**)
- $(\varphi \leftrightarrow \psi)$ is short for $((\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi))$ (**equivalence**)
- parentheses omitted when not necessary
- (\neg) binds more tightly than binary connectives
- (\wedge) binds more tightly than (\vee) than (\rightarrow) than (\leftrightarrow)

Definition (propositional valuation)

A **valuation** of propositions A is a function $v : A \rightarrow \{0, 1\}$.

Define the notation $v \models \varphi$ (v **satisfies** φ ; v is a **model** of φ ; φ is **true** under v) for valuations v and formulae φ by

- $v \models \top$
- $v \not\models \perp$
- $v \models a$ iff $v(a) = 1$, for $a \in A$.
- $v \models \neg\varphi$ iff $v \not\models \varphi$
- $v \models \varphi \vee \psi$ iff $v \models \varphi$ or $v \models \psi$
- $v \models \varphi \wedge \psi$ iff $v \models \varphi$ and $v \models \psi$

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

- A propositional formula ϕ is **satisfiable** if there is at least one valuation v so that $v \models \phi$.
- Otherwise it is **unsatisfiable**.
- A propositional formula ϕ is **valid** or a **tautology** if $v \models \phi$ for all valuations v .
- A propositional formula ψ is a **logical consequence** of a propositional formula ϕ , written $\phi \models \psi$, if $v \models \psi$ for all valuations v with $v \models \phi$.
- Two propositional formulae ϕ and ψ are **logically equivalent**, written $\phi \equiv \psi$, if $\phi \models \psi$ and $\psi \models \phi$.

Question: How to phrase these in terms of **models**?

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary



- A propositional formula that is a proposition a or a negated proposition $\neg a$ for some $a \in A$ is a **literal**.
- A formula that is a disjunction of literals is a **clause**. This includes **unit clauses** / consisting of a single literal, and the **empty clause** \perp consisting of zero literals.

Normal forms: NNF, CNF, DNF



Transitions for state sets described by propositions A can be concisely represented as **operators** or **actions** $\langle \chi, e \rangle$ where

- the **precondition** χ is a propositional formula over A describing the set of states in which the transition can be taken (states in which a transition starts), and
- the **effect** e describes how the resulting successor states are obtained from the state where the transitions is taken (where the transition goes).

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Blocks world operators

To model blocks world operators conveniently, we use auxiliary state variables *A-clear*, *B-clear*, and *C-clear* to denote that there is nothing on top of a given block.

Then blocks world operators can be modeled as:

- $\langle A\text{-clear} \wedge A\text{-on-}T \wedge B\text{-clear}, A\text{-on-}B \wedge \neg A\text{-on-}T \wedge \neg B\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}T \wedge C\text{-clear}, A\text{-on-}C \wedge \neg A\text{-on-}T \wedge \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}B, A\text{-on-}T \wedge \neg A\text{-on-}B \wedge B\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}C, A\text{-on-}T \wedge \neg A\text{-on-}C \wedge C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}B \wedge C\text{-clear}, A\text{-on-}C \wedge \neg A\text{-on-}B \wedge B\text{-clear} \wedge \neg C\text{-clear} \rangle$
- $\langle A\text{-clear} \wedge A\text{-on-}C \wedge B\text{-clear}, A\text{-on-}B \wedge \neg A\text{-on-}C \wedge C\text{-clear} \wedge \neg B\text{-clear} \rangle$
- ...

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (effects)

(Deterministic) **effects** are recursively defined as follows:

- If $a \in A$ is a state variable, then a and $\neg a$ are effects (**atomic effect**).
- If e_1, \dots, e_n are effects, then $e_1 \wedge \dots \wedge e_n$ is an effect (**conjunctive effect**).
The special case with $n = 0$ is the empty effect \top .
- If χ is a propositional formula and e is an effect, then $\chi \triangleright e$ is an effect (**conditional effect**).

Atomic effects a and $\neg a$ are best understood as assignments $a := 1$ and $a := 0$, respectively.

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

$\chi \triangleright e$ means that change e takes place if χ is true in the current state.

Example

Increment 4-bit number $b_3b_2b_1b_0$ represented as four state variables b_0, \dots, b_3 :

$$\begin{aligned} &(\neg b_0 \triangleright b_0) \wedge \\ &((\neg b_1 \wedge b_0) \triangleright (b_1 \wedge \neg b_0)) \wedge \\ &((\neg b_2 \wedge b_1 \wedge b_0) \triangleright (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ &((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \triangleright (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0)) \end{aligned}$$

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (changes caused by an operator)

For each effect e and state s , we define the **change set** of e in s , written $[e]_s$, as the following set of literals:

- $[a]_s = \{a\}$ and $[\neg a]_s = \{\neg a\}$ for atomic effects a , $\neg a$
- $[e_1 \wedge \dots \wedge e_n]_s = [e_1]_s \cup \dots \cup [e_n]_s$
- $[\chi \triangleright e]_s = [e]_s$ if $s \models \chi$ and $[\chi \triangleright e]_s = \emptyset$ otherwise

Definition (applicable operators)

Operator $\langle \chi, e \rangle$ is **applicable in a state s** iff $s \models \chi$ and $[e]_s$ is consistent (i. e., does not contain two complementary literals).

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (successor state)

The **successor state** $app_o(s)$ of s with respect to operator $o = \langle \chi, e \rangle$ is the state s' with $s' \models [e]_s$ and $s'(v) = s(v)$ for all state variables v not mentioned in $[e]_s$.

This is defined only if o is applicable in s .

Example

Consider the operator $\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle$ and the state $s = \{a \mapsto 1, b \mapsto 1, c \mapsto 1, d \mapsto 1\}$.

The operator is applicable because $s \models a$ and $[\neg a \wedge (\neg c \triangleright \neg b)]_s = \{\neg a\}$ is consistent.

Applying the operator results in the successor state $app_{\langle a, \neg a \wedge (\neg c \triangleright \neg b) \rangle}(s) = \{a \mapsto 0, b \mapsto 1, c \mapsto 1, d \mapsto 1\}$.

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (deterministic planning task)

A **deterministic planning task** is a 4-tuple $\Pi = \langle A, I, O, \gamma \rangle$ where

- A is a finite set of **state variables** (propositions),
- I is a valuation over A called the **initial state**,
- O is a finite set of **operators** over A , and
- γ is a formula over A called the **goal**.

Note:

- When we talk about deterministic planning tasks, we usually omit the word “deterministic”.
- When we will talk about nondeterministic planning tasks later, we will explicitly qualify them as “nondeterministic”.

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary

Definition (induced transition system of a planning task)

Every planning task $\Pi = \langle A, I, O, \gamma \rangle$ induces a corresponding deterministic transition system $\mathcal{T}(\Pi) = \langle S, L, T, s_0, S_\star \rangle$:

- S is the set of all valuations of A ,
- L is the set of operators O ,
- $T = \{ \langle s, o, s' \rangle \mid s \in S, o \text{ applicable in } s, s' = \text{app}_o(s) \}$,
- $s_0 = I$, and
- $S_\star = \{ s \in S \mid s \models \gamma \}$



- Terminology for transitions systems is also applied to the planning tasks that induce them.
- For example, when we speak of the **states of Π** , we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a goal path of $\mathcal{T}(\Pi)$ is called a **plan** of Π .

By **planning**, we mean the following two algorithmic problems:

Definition (satisficing planning)

Given: a planning task Π

Output: a plan for Π , or **unsolvable** if no plan for Π exists

Definition (optimal planning)

Given: a planning task Π

Output: a plan for Π with minimal length among all plans for Π , or **unsolvable** if no plan for Π exists

Transition
systems

Planning
tasks

State variables

Logic

Operators

Tasks

Summary



- **Transition systems** are (typically huge) directed graphs that encode how the state of the world can change.
- **Planning tasks** are compact representations for transition systems, suitable as input for planning algorithms.
- Planning tasks are based on concepts from **propositional logic**, enhanced to model state change.
- **States** of planning tasks are propositional valuations.
- **Operators** of planning tasks describe **when** (precondition) and **how** (effect) to change the current state of the world.
- In **satisficing planning**, we must find a solution to planning tasks (or show that no solution exists).
- In **optimal planning**, we additionally guarantee that generated solutions are of the shortest possible length.

Transition
systems

Planning
tasks

Summary