Exercise 3.1 (S5: Axioms and frame properties I, 6 points)
A Kripke frame $\mathcal{F} = \langle S, R \rangle$ is defined exactly like a Kripke model $\langle S, R, V \rangle$, but without the valuation $V$. The set of all models over $\langle S, R \rangle$ is the set of all models $\langle S, R, V \rangle$ where $V$ is any propositional valuation. A formula is valid in a frame $\mathcal{F}$, if it is valid in all models over $\mathcal{F}$. It is valid in a class of frames, if it is valid in each frame in that class. We say that an axiom defines a class of frames if the axiom is valid exactly in this class of frames. Show that

(a) the axiom $T$ defines the class of reflexive frames,

(b) the axiom $4$ defines the class of transitive frames,

(c) the axiom $5$ defines the class of Euclidean frames.

Note: You might be able to re-use parts of your solutions for Exercise 2.2.

Exercise 3.2 (S5: Axioms and frame properties II, 3 points)
Show that the class of frames that is defined by the axioms $K$, $T$ and $5$ is the same as the class of frames that is defined by the axioms $K$, $T$, $4$ and $5$. You can use the correspondences of frame properties to axioms from the previous exercise.

Exercise 3.3 (S5: Deriving theorems, 1+1+1 points)
Derive the following S5 theorems. Recall that a derivation is a finite sequence of formulas, such that each formula is either an instance of one of the axioms, an instance of a propositional tautology, or the result of the application of one of the rules (necessitation, modus ponens) on previous formulas.

(a) $K_a(p \rightarrow p)$

(b) $K_a p \rightarrow \hat{K}_a p$

(c) $K_a K_b p \rightarrow K_a p$