Exercise 1.1 (Cheryl’s Birthday, 3+3+3)

Have a look at the following riddle, which was given to 14 and 15 year old students in Singapore and gained some media attention last year:

Albert and Bernard just became friends with Cheryl, and they want to know when her birthday is. Cheryl gives them a list of 10 possible dates: May 15, May 16, May 19, June 17, June 18, July 14, July 16, August 14, August 15, and August 17.

Cheryl then tells Albert and Bernard separately the month and the day of her birthday respectively.

Albert: I don’t know when Cheryl’s birthday is, but I know that Bernard does not know too.
Bernard: At first I didn’t know when Cheryl’s birthday is, but I know now.
Albert: Then I also know when Cheryl’s birthday is.

(a) What is “commonly known” between Albert and Bernard before they start their dialogue? Characterize the situation using $L_K$ formulas.

(b) Model the announcements of Albert and Bernard as $L_K$ formulas. Since we cannot model temporal statements, you are allowed to omit the first part of Bernard’s announcement (“at first I didn’t know when Cheryl’s birthday is”).

(c) When is Cheryl’s birthday? Explain your answer.

Exercise 1.2 (Hanabi, 1+1+1)

Model the following statements about a Hanabi instance in our simplified setting (two players, two suits two numbers, and one card per player) as $L_K$ formulas:

(a) Agent $a$ does not know the color of his own card.

(b) Agent $b$ knows that his own card is a 1 and that agent $a$ knows its color.

(c) Agent $a$ considers it possible that agent $b$ knows both color and number of his (agent $b$’s) own card.