4.1 Motivation

So far. Only public announcements.
How can we model "private announcements", etc...?

Example 1  a and b both don't know the value of proposition p. This is common knowledge among them. In fact p is true. Then a receives a letter containing the value of p and reads it. Agent b observes a reading the letter and knows that it is about p, but b does not learn the value of p.

Then, After = Before ⊗ Read
For an appropriate definition of ⊗:
Definition 1 \( \otimes \) is a restricted modal update with component worlds \((s,e)\) only present if \((M,s) \models \text{pre}(e)\)

Figure 4: Before \( \otimes \) Read

\((s_1,e_1) \sim_b (s_2,e_2)\) because \(s_1 \sim_b s_2\) and \(e_1 \sim_b e_2\).

\((s_1,e_2)\) and \((s_2,e_1)\) were eliminated because \(e_2\) cannot be applied in \(s_1\) and \(e_1\) cannot be applied in \(s_2\).

4.2 Action models

Definition 2 Let \( \mathcal{L} \) be any logical language for a set of agents \( A \) and a set of atoms \( P \). Then an \( S_5 \) action model \( m \) is a structure \( \langle S, \sim, \text{pre} \rangle \) s.t.:

- \( S \) is the domain of action events,
- \( \forall a \in A, \sim_a \) is an equivalence relation on \( S \),
- \( \text{pre}: S \to \mathcal{L} \) is the precondition function that assigns a precondition \( \text{pre}(s) \in \mathcal{L} \),
- \( \forall s \in S \)

A pointed action model is such a structure \((M,s)\) with \( s \in S \)

4.3 Syntax of action model logic

Definition 3 Given \( A \) and \( P \), the language of action model logic \( \mathcal{L}_{KC\otimes} \) is the union of the formulas \( \phi \in \mathcal{L}_{KC\otimes}^{\text{stat}}(A,P) \) and the actions \( \alpha \in \mathcal{L}_{KC\otimes}^{\text{act}}(A,P) \) defined by:

\[\phi ::= p | \neg p | (\phi \land \phi) | K_a \phi | C_B \phi | [\alpha] \phi\]

\[\alpha ::= (M,s) | \alpha \lor \alpha\]

where \( p \in P \), \( a \in A \), \( B \subseteq A \) and \((M,s)\) is a pointed action model with:

1. finite domain \( S \),
2. such that \( \forall t \in S \), the precondition \( \text{pre}(t) \) is a \( \mathcal{L}_{KC\otimes}^{\text{stat}}(A,P) \) formula that has already been constructed in a previous step of the induction
Example 2 (Public announcements)
Public announcements can be viewed as action models.

\[
\text{pre} : \phi
\]

Figure 5: Action model for the public announcement of \( \phi \)

\[
\langle \alpha \rangle := \neg[\alpha]\neg\phi
\]
\[
M = \bigcup_{s \in S}(M, s)
\]

Definition 4 (Composition)
Let \( m = \langle S, \sim, \text{pre} \rangle \) and \( m' = \langle S', \sim', \text{pre}' \rangle \) be action models in \( L_{\text{actKC}}(A, P) \). Then their composition \( (m; m') \) is the action model \( \langle S'', \sim'', \text{pre}'' \rangle \) s.t. :

\[
S = S \times S'
\]
\[
(s, s') \sim''_a (t, t') \text{ iff } s \sim_a t \text{ and } s' \sim'_a t'
\]
\[
\text{pre}((s, s')) = (M, s)\text{pre}'(s')
\]

For pointed action models: \( ((m, t); (m', t')) = ((m; m'), (t; t')) \)

4.4 Semantics of action model logic

Definition 5 Let \( m = \langle S, \sim, V \rangle \) be an epistemic models and let \( m' = \langle S', \sim', \text{pre} \rangle \) be an action model. Then the product update of \( m \otimes m' \) is the model \( m'' = \langle S'', \sim'', V'' \rangle \), where:

\[
S'' = \{(s, s') \in S \times S' | m, s \models \text{pre}'(s')\}
\]
\[
(s, s') \sim''_a (t, t') \text{ iff } s \sim_a t \text{ and } s' \sim'_a t' \text{ for } a \in A
\]
\[
(s, s') \in V' \iff s \in V
\]

Example 3 (Before, \( S_1 \)) \( \otimes \) (Read, \( e_1 \)) = (After, \( (s_1, e_1) \))

Definition 6 (Semantics of formulas and actions)
Let \( (m, s) \) be an epistemic state and \( m' = \langle S', \sim', \text{pre} \rangle \) an action model, \( \phi \in L_{\text{statKC}}(A, P) \) and \( \alpha \in L_{\text{actKC}}(A, P) \). Then:

\[
m, s \models \phi \iff \exists \phi \models \phi \land \phi \models \Box \phi \models \Box \phi
\]
\[
m, s \models \alpha \iff \forall m'', s'' : (m, s)[\alpha](m'', s'') \text{ implies } m'', s'' \models \phi \text{ where }
\]
\[
(m, s)[[m', s']](m'', s'') \iff (m, s) \models \text{pre}'(s') \text{ and } (m'', s'') = (m \otimes, m', (s, s')) \text{ and }
\]
\[
[\alpha \cup \alpha'] = [\alpha] \cup [\alpha']
\]

For \( \alpha = (m', s') : m, s \models [m', s']\phi \iff (m, s) \models \text{pre}'(s') \text{ implies } (m \otimes m', (s, s')) \models \phi \)
Remark: Compare to semantics of $[\phi] \psi$ and $(\phi) \psi$ in PA.

Proposition: Let $(m, s), (m', s') \in \mathcal{L}_{KC}^{act}(A, P)$ and $\phi \in \mathcal{L}_{KC}^{stat}(A, P)$. Then: $
[(m, s); (m', s')]/\phi$ is equivalent to $[(m, s)][(m', s')]/\phi$