Exercise Sheet 7
Due: Friday, December 9th, 2016

Exercise 7.1 (Abstraction heuristics, 4+4 points)

A state of a 15-puzzle planning task is given as a permutation \( \langle b, t_1, \ldots, t_{15} \rangle \) of \{1, \ldots, 16\}, where \( b \) denotes the empty tile (blank) and all other components denote the positions of the tiles.

Let \( T^1 = \{ t_1, \ldots, t_n \}, T^2 = \{ t_1', \ldots, t_m' \} \) with \( 1 \leq n, m \leq 14 \) be a partitioning of \{1, \ldots, 15\} (i.e., \( T^1 \cup T^2 = \{ t_1, \ldots, t_{15} \} \) and \( T^1 \cap T^2 = \emptyset \)). Consider the following abstractions:

- \( \alpha_1((b, t_1, \ldots, t_{15})) = (b, t_1', \ldots, t_{m}') \)
- \( \alpha_2((b, t_1, \ldots, t_{15})) = (b, t_1, \ldots, t_m') \)
- \( \alpha_3((b, t_1, \ldots, t_{15})) = (t_1, \ldots, t_{m}') \)
- \( \alpha_4((b, t_1, \ldots, t_{15})) = (t_1', \ldots, t_{m}') \)

For \( 1 \leq i \leq 4 \), the heuristic estimates of \( h_i \) are equal to lengths of optimal plans in the respective abstractions (e.g., \( h_i(s) = h^*(\alpha_i(s)) \)). Show that:

(a) \( h_1 + h_2 \) is not admissible.

(b) \( h_3 + h_4 \) is admissible.

*Hint*: A heuristic is admissible if it is goal-aware and consistent.

You may and should solve the exercise sheets in groups of two. Please state both names on your solution.