Introduction
ASP: Background

- **Answer set semantics**: a formalization of negation as failure in logic programming (Prolog)
- Several formal semantics: well-founded semantics, perfect-model semantics, inflationary semantics, ...
- Can be viewed as a simpler variant of default logic
ASP: Negation as failure

- Another interpretation for negation: $\text{not} \, x \equiv \text{"It cannot be shown that } x \text{ is true"}$
- For example, you are innocent until proven guilty

Example

\[
\text{innocent} \leftarrow \text{not guilty}.
\]
ASP: Declarative problem solving

- What is the problem? instead of: How to solve the problem?
- Outsourcing the computation part to an external solver
Answer Sets
Normal logic programs I

Let $\mathcal{A}$ be a set of first-order atoms.

**Rules:**

\[ a \leftarrow b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_k \]

where $\{a, b_1, \ldots, b_m, c_1, \ldots, c_k\} \subseteq \mathcal{A}$

- Meaning similar to default logic:
  - If
    1. we have derived $b_1, \ldots, b_m$ and
    2. cannot derive any of $c_1, \ldots, c_k$,
  - then derive $a$.

- Rules without right-hand side (facts): $a \leftarrow$
- Rules without left-hand side (constraints):
  \[ \leftarrow b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_k \]
Normal logic programs II

Let $\mathcal{A}$ be a set of first-order atoms.

Rules:

\[ a \leftarrow b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_k \]

where $\{a, b_1, \ldots, b_m, c_1, \ldots, c_k\} \subseteq \mathcal{A}$

- $a$ is called the head of the rule, denoted by $\text{head}(r)$.
- The literals $b_1, \ldots, b_m$ form the positive body of $r$, denoted by $\text{body}^+(r)$.
- The literals $\neg c_1, \ldots, \neg c_k$ form the negative body of $r$, denoted by $\text{body}^-(r)$.
- The body of $r$ is the union of positive and negative body: $\text{body}(r) = \text{body}^+(r) \cup \text{body}^-(r)$. 
Normal logic programs: Example

Example

\[
\begin{align*}
\text{bird}(X) & \leftarrow \text{eagle}(X) \\
\text{bird}(X) & \leftarrow \text{penguin}(X) \\
\text{fly}(X) & \leftarrow \text{bird}(X), \text{not} \ \text{nonfly}(X) \\
\text{nonfly}(X) & \leftarrow \text{penguin}(X) \\
\text{eagle}(\text{eddy}) & \leftarrow \\
\text{penguin}(\text{tweety}) & \leftarrow
\end{align*}
\]
Let $P$ be a normal logic program, i.e., a finite set of rules as described above.

- The **Herbrand universe** (symb. $U_P$) of $P$ is the set of ground terms constructed from the function symbols and constants in $P$.

- The **Herbrand base** of $P$ (symb. $B_P$) is the set of ground atoms constructed from predicate symbols and ground terms from the Herbrand universe.

- From now on, a program will refer to the set of its grounded rules.

- The set of atoms in $P$ is denoted by $\text{atoms}(P)$.
Herbrand base and grounded rules

Example

\[
\begin{align*}
\text{bird}(\text{eddy}) & \leftarrow \text{eagle}(\text{eddy}) \\
\text{bird}(\text{tweety}) & \leftarrow \text{eagle}(\text{tweety}) \\
\text{bird}(\text{eddy}) & \leftarrow \text{penguin}(\text{eddy}) \\
\text{bird}(\text{tweety}) & \leftarrow \text{penguin}(\text{tweety}) \\
\text{fly}(\text{eddy}) & \leftarrow \text{bird}(\text{eddy}), \neg \text{nonfly}(\text{eddy}) \\
\text{fly}(\text{tweety}) & \leftarrow \text{bird}(\text{tweety}), \neg \text{nonfly}(\text{tweety}) \\
\text{nonfly}(\text{eddy}) & \leftarrow \text{penguin}(\text{eddy}) \\
\text{nonfly}(\text{tweety}) & \leftarrow \text{penguin}(\text{tweety}) \\
\text{eagle}(\text{eddy}) & \leftarrow \\
\text{penguin}(\text{tweety}) & \leftarrow
\end{align*}
\]
Satisfaction

A Herbrand interpretation is a subset $X$ of the Herbrand base.
A Herbrand interpretation is a subset $X$ of the Herbrand base.

Satisfaction relation:

- $X \models a$ if $a \in X$.
- $X \models r$ if $\{b_1, \ldots, b_m\} \not\subseteq X$ or $\{a, c_1, \ldots, c_n\} \cap X \neq \emptyset$, where $r = a \leftarrow b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_k$.
- $X \models P$ if $X \models r$ for each $r \in P$. 
**Satisfaction**

A **Herbrand interpretation** is a subset $X$ of the Herbrand base.

**Satisfaction relation:**

- $X \models a$ if $a \in X$.
- $X \models r$ if $\{b_1, \ldots, b_m\} \not\subseteq X$ or $\{a, c_1, \ldots, c_n\} \cap X \neq \emptyset$, where $r = a \leftarrow b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_k$.
- $X \models P$ if $X \models r$ for each $r \in P$.

**Idea**

Idea: “models” as interpretations that are satisfying, stable, and supported.
Positive \textit{(not-free)} logic programs

Definition (Answer set)

Let $P$ be a logic program without $\textbf{not}$, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) answer set of $P$ if it is the least fixpoint of the operator:

$$\Gamma_P(X) = \{a: \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X\}.$$
**Positive (not-free) logic programs**

**Definition (Answer set)**

Let $P$ be a logic program without **not**, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) **answer set** of $P$ if it is the least fixpoint of the operator:

$$\Gamma_P(X) = \{a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X\}.$$

**Example**

$$P = \left\{ \begin{array}{l}
a \leftarrow b, \\
d \leftarrow f, \\
b \leftarrow, \\
d \leftarrow b, \\
c \leftarrow d, \\
e \leftarrow f \end{array} \right\}$$
Positive \textit{(not-free)} logic programs

\section*{Definition (Answer set)}

Let $P$ be a logic program \textbf{without not}, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) \textbf{answer set} of $P$ if it is the least fixpoint of the operator:

$$\Gamma_P(X) = \{ a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X \}.$$ 

\section*{Example}

$$P = \left\{ a \leftarrow b, \quad d \leftarrow f, \quad b \leftarrow, \quad d \leftarrow b, \quad c \leftarrow d, \quad e \leftarrow f \right\}$$

$$\Gamma^0 = \emptyset.$$
Positive (not-free) logic programs

Definition (Answer set)

Let \( P \) be a logic program without not, \( X \subseteq \text{atoms}(P) \).
\( X \) is the (unique) answer set of \( P \) if it is the least fixpoint of the operator:

\[
\Gamma_P(X) = \{a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X\}.
\]

Example

\[
P = \left\{ \begin{array}{ccc}
a & \leftarrow & b, \\
d & \leftarrow & f, \\
d & \leftarrow & b, \\
b & \leftarrow,
\end{array} \right\}
\]

\[
\Gamma^0 = \emptyset, \quad \Gamma^1 = \Gamma(\emptyset) = \{b\}.
\]
Positive (not-free) logic programs

Definition (Answer set)

Let $P$ be a logic program without not, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) answer set of $P$ if it is the least fixpoint of the operator:

$$\Gamma_P(X) = \{a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X\}.$$

Example

$$P = \begin{cases} a \leftarrow b, & d \leftarrow f, & b \leftarrow, \\ d \leftarrow b, & c \leftarrow d, & e \leftarrow f \end{cases}$$

$$\Gamma^0 = \emptyset, \quad \Gamma^1 = \Gamma(\emptyset) = \{b\}, \quad \Gamma^2 = \Gamma(\Gamma^1) = \{b, d, a\},$$
Positive (*not-free*) logic programs

**Definition (Answer set)**

Let $P$ be a logic program without *not*, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) answer set of $P$ if it is the least fixpoint of the operator:

$$\Gamma_P(X) = \{a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X\}.$$

**Example**

$$P = \left\{ \begin{array}{ll} a \leftarrow b, & d \leftarrow f, \ b \leftarrow, \\ d \leftarrow b, & c \leftarrow d, \ e \leftarrow f \end{array} \right\}$$

$$\Gamma^0 = \emptyset, \quad \Gamma^1 = \Gamma(\emptyset) = \{b\}, \quad \Gamma^2 = \Gamma(\Gamma^1) = \{b, d, a\}, \quad \Gamma^3 = \Gamma(\Gamma^2) = \{b, d, a, c\}.$$
Positive \textit{(not-free)} logic programs

**Definition (Answer set)**

Let $P$ be a logic program \textbf{without not}, $X \subseteq \text{atoms}(P)$. $X$ is the (unique) \textbf{answer set} of $P$ if it is the least fixpoint of the operator:

$$
\Gamma_P(X) = \{ a : \exists r = a \leftarrow b_1, \ldots, b_m \in P \text{ with } \{b_1, \ldots, b_m\} \subseteq X \}.
$$

**Example**

$$
P = \begin{cases}
    a \leftarrow b, & d \leftarrow f, & b \leftarrow, \\
    d \leftarrow b, & c \leftarrow d, & e \leftarrow f
\end{cases}
$$

$$
\Gamma^0 = \emptyset, \quad \Gamma^1 = \Gamma(\emptyset) = \{b\}, \quad \Gamma^2 = \Gamma(\Gamma^1) = \{b, d, a\}, \quad \Gamma^3 = \Gamma(\Gamma^2) = \{b, d, a, c\}, \quad \Gamma^4 = \Gamma(\Gamma^3) = \{b, d, a, c\} = \Gamma^3
$$
Gelfond-Lifschitz reduct

Definition (Reduct)

The **reduct** of a program $P$ with respect to a set of atoms $X \subseteq \text{atoms}(P)$ is defined as:

$$P^X := \{ \text{head}(r) \leftarrow \text{body}^+(r) : r \in P, c \notin X \text{ for each not } c \in \text{body}^-(r) \}$$
Gelfond-Lifschitz reduct

**Definition (Reduct)**

The reduct of a program $P$ with respect to a set of atoms $X \subseteq \text{atoms}(P)$ is defined as:

$$P^X := \{ \text{head}(r) \leftarrow \text{body}^+(r) : r \in P, c \notin X \text{ for each } \text{not} c \in \text{body}^-(r) \}$$

That is, given $X$,

- ... delete all rules whose negative part contradicts $X$
- ... remove all negated atoms from the remaining rules
Gelfond-Lifschitz reduct

Definition (Reduct)

The reduct of a program $P$ with respect to a set of atoms $X \subseteq \text{atoms}(P)$ is defined as:

$$P^X := \{ \text{head}(r) \leftarrow \text{body}^+(r) : r \in P, \quad c \notin X \text{ for each not } c \in \text{body}^-(r)\}$$

That is, given $X$,

- ... delete all rules whose negative part contradicts $X$
- ... remove all negated atoms from the remaining rules

Definition (Answer set)

$X \subseteq \text{atoms}(P)$ is an answer set of $P$ if $X$ is an answer set of $P^X$. 
Answer sets: Examples

Example

\[
a \leftarrow \text{not } b, \quad b \leftarrow \text{not } a, \\
c \leftarrow a, \quad d \leftarrow b.
\]
Answer sets: Examples

Example

\[
\begin{align*}
a &\leftarrow \neg b, & b &\leftarrow \neg a, \\
c &\leftarrow a, & d &\leftarrow b.
\end{align*}
\]

Example

\[
\begin{align*}
a &\leftarrow \neg b, & b &\leftarrow \neg a, \\
b &\leftarrow a, & c &\leftarrow b
\end{align*}
\]

Notice X can satisfy all rules, but may not be an answer set!
## Answer sets: Examples

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \leftarrow \text{not} b$, $b \leftarrow \text{not} a$, $c \leftarrow a$, $d \leftarrow b$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \leftarrow \text{not} b$, $b \leftarrow \text{not} a$, $b \leftarrow a$, $c \leftarrow b$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \leftarrow b$, $b \leftarrow a$</td>
</tr>
</tbody>
</table>
Answer sets: Examples

Example

\[ a \leftarrow \neg b, \quad b \leftarrow \neg a, \quad c \leftarrow a, \quad d \leftarrow b. \]

Example

\[ a \leftarrow \neg b, \quad b \leftarrow \neg a, \quad b \leftarrow a, \quad c \leftarrow b \]

Example

\[ a \leftarrow b, \quad b \leftarrow a \]
Some properties I

**Proposition**

*If an atom* \( a \) *belongs to an answer set of a logic program* \( P \), *then* \( a \) *is the head of one of the rules of* \( P \).*
Some properties I

**Proposition**

If an atom $a$ belongs to an answer set of a logic program $P$, then $a$ is the head of one of the rules of $P$.

**Proposition**

Each answer set of a normal logic program $P$ is a minimal model of $P$, i.e., it satisfies all rules in $P$ and there is no proper subset of $P$ satisfying all rules in $P$.

**Notice:** The converse is not true: not each minimal model is an answer set.
Some properties II

Proposition

Let $F$ be a set of (non-constraint) rules and $G$ be a set of constraints. A set of atoms $X$ is an answer set of $F \cup G$ iff it is an answer set of $F$ that satisfies $G$. 
Some properties II

Proposition

Let $F$ be a set of (non-constraint) rules and $G$ be a set of constraints. A set of atoms $X$ is an answer set of $F \cup G$ iff it is an answer set of $F$ that satisfies $G$.

Proof.

$F \subseteq F \cup G$ implies $F^X \subseteq (F \cup G)^X$ and hence $\text{lfp}_\Gamma(F^X) \subseteq \text{lfp}_\Gamma((F \cup G)^X))$.

$\Rightarrow$: Assume $X$ is an answer set of $F \cup G$, hence $X = \text{lfp}_\Gamma((F \cup G)^X)$ and $X \models G$. Since $G$ contains constraints only, it follows that each $a \in X$ is the head of some rule in $F$. Hence, $X \subseteq \text{lfp}_\Gamma(F^X)$, and thus $X$ is an answer set of $F$ that satisfies $G$. 

$\Leftarrow$: Similar.
Some properties II

Proposition

Let $F$ be a set of (non-constraint) rules and $G$ be a set of constraints. A set of atoms $X$ is an answer set of $F \cup G$ iff it is an answer set of $F$ that satisfies $G$.

Proof.

$F \subseteq F \cup G$ implies $F^X \subseteq (F \cup G)^X$ and hence $\text{lfp}_\Gamma(F^X) \subseteq \text{lfp}_\Gamma((F \cup G)^X))$.

$\Rightarrow$: Assume $X$ is an answer set of $F \cup G$, hence $X = \text{lfp}_\Gamma((F \cup G)^X)$ and $X \models G$. Since $G$ contains constraints only, it follows that each $a \in X$ is the head of some rule in $F$. Hence, $X \subseteq \text{lfp}_\Gamma(F^X)$, and thus $X$ is an answer set of $F$ that satisfies $G$.

$\Leftarrow$: Similar.
Complexity: Existence of answer sets is NP-complete

1 Membership in NP: Guess $X \subseteq \text{atoms}(P)$ (nondet. polytime), compute $P^X$, compute its closure, compare to $X$ (everything det. polytime).
Complexity: Existence of answer sets is NP-complete

1 **Membership in NP**: Guess $X \subseteq \text{atoms}(P)$ (nondet. polytime), compute $P^X$, compute its closure, compare to $X$ (everything det. polytime).

2 **NP-hardness**: Reduction from 3SAT: an answer set exists iff the following clauses are satisfiable:

$$p \leftarrow \neg \hat{p}. \quad \hat{p} \leftarrow \neg p.$$ 

for every propositional variable $p$ occurring in the clauses.
Complexity: Existence of answer sets is NP-complete

1. **Membership in NP**: Guess $X \subseteq \text{atoms}(P)$ (\textbf{nondet. polytime}), compute $P^X$, compute its closure, compare to $X$ (everything \textbf{det. polytime}).

2. **NP-hardness**: Reduction from 3SAT: an answer set exists iff the following clauses are satisfiable:

   $$p \leftarrow \neg \hat{p}.$$  
   $$\hat{p} \leftarrow \neg p.$$

   for every propositional variable $p$ occurring in the clauses, and

   $$\leftarrow \neg l'_1, \neg l'_2, \neg l'_3$$

   for every clause $l_1 \lor l_2 \lor l_3$, where $l'_i = p$ if $l_i = p$ and $l'_i = \hat{p}$ if $l_i = \neg p$. 
Difference to Propositional Logic

- The **ancestor** relation is the transitive closure of the **parent** relation.

- Transitive closure **cannot be** (concisely) represented in propositional/predicate logic.

\[
\begin{align*}
\text{par}(X,Y) & \rightarrow \text{anc}(X,Y) \\
\text{par}(X,Z) \land \text{anc}(Z,Y) & \rightarrow \text{anc}(X,Y)
\end{align*}
\]

The above formulae only guarantee that **anc** is a **superset** of the transitive closure of **par**.

- For transitive closure one needs the **minimality condition** in some form: nonmonotonic logics, fixpoint logics, ...
Stratification

The reason for multiple answer sets is the fact that \( a \) may depend on \( b \) and simultaneously \( b \) may depend on \( a \). The lack of this kind of circular dependencies makes reasoning easier.

Definition

A logic program \( P \) is **stratified** if \( P \) can be partitioned to \( P = P_1 \cup \cdots \cup P_n \) so that for all \( i \in \{1, \ldots, n\} \) and \((a \leftarrow b_1, \ldots, b_m, \text{not} c_1, \ldots, \text{not} c_k) \in P_i\),

1. there is no \( \text{not} a \) in \( P_i \) and
2. there are no occurrences of \( a \) anywhere in \( P_1 \cup \cdots \cup P_{i-1} \).
Stratification

**Theorem**

*A stratified program $P$ has exactly one answer set. The unique answer set can be computed in polynomial time.*
Stratification

Theorem

A stratified program $P$ has exactly one answer set. The unique answer set can be computed in polynomial time.

Example

Our earlier examples with more than one or no answer sets:

$$P_3 = \{p \leftarrow \text{not} p\}$$
$$P_4 = \{p \leftarrow \text{not} q, \quad q \leftarrow \text{not} p\}$$
AnsProlog and ASP Tools
Programs for Reasoning with Answer Sets

- smodels (Niemelä & Simons), dlv (Eiter et al.), clasp (Schaub et al.), ...

- Schematic input:

  
  \[
  \begin{align*}
  p(X) & \leftarrow \text{not} \quad q(X). \\
  q(X) & \leftarrow \text{not} \quad p(X). \\
  r(a). \\
  r(b). \\
  r(c). \\
  \text{anc}(X,Y) & \leftarrow \text{par}(X,Y). \\
  \text{anc}(X,Y) & \leftarrow \text{par}(X,Z), \quad \text{anc}(Z,Y). \\
  \text{par}(a,b). \quad \text{par}(a,c). \quad \text{par}(b,d). \\
  \text{female}(a). \\
  \text{male}(X) & \leftarrow \text{not}(\text{female}(X)). \\
  \text{forefather}(X,Y) & \leftarrow \text{anc}(X,Y), \quad \text{male}(X). 
  \end{align*}
  \]
Propositions are any combination of lowercase letters.

Variables are any combination of letters starting with an uppercase letter.

Write ":-" instead of ←.

Integers can be used and so can arithmetic operations (+, −, *, /, %).

Negation as failure is denoted by not.

Strong negation is denoted by ¬.

#const n = ... statements can be used to define constants.

The #hide/#show statements can be used to influence which iterals are shown in the solution.
AnsProlog: Choice functions

- The literal \{b_1; \ldots ; b_m\} is true iff any subset of the set \{b_1, \ldots, b_m\} is true.
AnsProlog: Choice functions

- The literal \{b_1; \ldots ; b_m\}
is true iff any subset of the set \{b_1, \ldots , b_m\} is true.

**Example**

Generate all interpretations over the atoms \(a(1), a(2), a(3)\):

\{ a(1); a(2); a(3) \}. 
AnsProlog: Choice functions

The literal \{b_1; \ldots; b_m\}

is true iff any subset of the set \{b_1, \ldots, b_m\} is true.

Example

Generate all interpretations over the atoms \(a(1), a(2), a(3)\):

\{ a(1); a(2); a(3) \}.

With strong negation:

\(-a(X) :- \text{not } a(X), X=1..3.\)

\{ a(1..3) \}. 
AnsProlog: Choice with cardinality

- The literal $l \{b_1; \ldots; b_m\} u$ is true iff at least $l$ and at most $u$ atoms (included) are true within the set $\{b_1, \ldots, b_m\}$.
AnsProlog: Choice with cardinality

- The literal $l \{b_1; \ldots; b_m\} u$ is true iff at least $l$ and at most $u$ atoms (included) are true within the set $\{b_1,\ldots,b_m\}$.

Example

Generate all interpretations over the atoms $a(1), a(2), a(3), b(1), b(2)$ that contain exactly 2 true atoms:

$$2 \{ a(1..3); b(1..2) \} 2.$$
AnsProlog: Choice with cardinality

- The literal \( l \{b_1; \ldots; b_m\} u \)
  is true iff at least \( l \) and at most \( u \) atoms (included) are true
  within the set \( \{b_1, \ldots, b_m\} \).

**Example**

Generate all interpretations over the atoms \( a(1), a(2), a(3), b(1), b(2) \)
that contain exactly 2 true atoms:

\[
2 \{ a(1..3); b(1..2) \} 2.
\]

Generate all interpretations over the atoms \( a(1), a(2), a(3), b(1), b(2), b(3) \)
that do not contain exactly 2 or more true atoms for the same predicate:

\[
\{ a(1..3); b(1..3) \}.
\]

\[
:- 2 \{ a(1..3) \} 3.
\]

\[
:- 2 \{ b(1..3) \} 3.
\]
AnsProlog: Domains of variables

- The domain of a variable must be known in order to avoid "unsafe"-error while the program is grounded.
- The domain can be set literal-wise, rule-wise, or program wise.
- For limiting the scope within a literal use the syntax: 
  \[ a(X) : \text{dom}(X) \quad \text{or} \quad a(X) : X=1..3 \]
AnsProlog: Domains of variables

- The domain of a variable must be known in order to avoid “unsafe”-error while the program is grounded.
- The domain can be set literal-wise, rule-wise, or program wise.
- For limiting the scope within a literal use the syntax:
  \[ a(X) : \text{dom}(X) \quad \text{or} \quad a(X) : X=1..3 \]

Example

\[
\begin{align*}
\text{num(0..10).} \\
\text{even(2*X) :- num(X), 2*X <=10.} \\
1 \{ a(X) : \text{even}(X) \} 1.
\end{align*}
\]

\#show a/1.
Example: Graph coloring

Example

```
#const n = 2.
c(1..n).
1 {color(X,I) : c(I)} 1 :- v(X).
:- color(X,I), color(Y,I), e(X,Y), c(I).

% Instance
v(1..4).
e(1,2).
e(1,3).
e(2,4).
e(3,4).
% e(2,3).

#show color/2.
```
Generate and test

ASP programs are often organized in a “generate-and-test” style: first describe candidate solutions, then rule out possible solutions by stating constraints.

Example

% n-Queens encoding %
#const n = 4.

% Generate possible positions %
1 { q(I,1..n) } 1 :- I = 1..n.

% Rule out attacking positions %
:- q(I1,J), q(I2,J), I1 != I2.
:- q(I,J), q(I+D,J+D), D = 1..n.
:- q(I,J), q(I+D,J-D), D = 1..n.
Generate and test

ASP programs are often organized in a “generate-and-test” style: first describe candidate solutions, then rule out possible solutions by stating constraints.

Example

% n-Queens encoding %
#const n = 4.

% Generate possible positions %
1 { q(I,1..n) } 1 :- I = 1..n.

% Rule out attacking positions %
:- q(I1,J), q(I2,J), I1 != I2.
:- q(I,J), q(I+D,J+D), D = 1..n.
:- q(I,J), q(I+D,J-D), D = 1..n.
Generate and test: Further example

**Problem:** In a graph find cliques of size $\geq n$
Generate and test: Further example

**Problem:** In a graph find cliques of size $\geq n$

**Example**

```prolog
#const n = 3.

dge(X,Y) :- edge(Y,X).
n {clique(X) : node(X)}.
:- clique(X), clique(Y), node(X), node(Y), X!=Y, not edge(X,Y).

% Instance %
node(1..5).
edge(1,2;4).
edge(2,3;4).
edge(3,4).
edge(4,2;5).
#show clique/1.
```

The language is even bigger than that! It includes

- Disjunction in the head
- Other operators: #sum,#min,#max,#even,#odd,#avg, ...
- Multi-criteria optimizations
- Heuristic optimizations
- ...

(More on that in the exercises!)
Literature

Michael Gelfond and Vladimir Lifschitz.  
The stable models semantics for logic programming.  

Francois Fages.  
Consistency of Clark’s completion and existence of stable models.  
Meth. of Logic in CS, p51-60, 1994.

Hudson Turner.  
Strong equivalence made easy: nested expressions and weight constraints.  
Literature

Martin Gebser and Benjamin Kaufmann and André Neumann and Torsten Schaub.

Conflict-Driven Answer Set Solving.

Ilkka Niemelä and Patrik Simons

Efficient Implementation of the Well-founded and Stable Model Semantics.