Motivation
Example TBox & ABox

- Male $\triangleq \neg$Female
- Human $\sqsubseteq$ Living_entity
- Woman $\triangleq$ Human $\sqcap$ Female
- Man $\triangleq$ Human $\sqcap$ Male
- Mother $\triangleq$ Woman $\sqcap \exists$has-child.Human
- Father $\triangleq$ Man $\sqcap \exists$has-child.Human
- Parent $\triangleq$ Father $\sqcup$ Mother
- Grandmother $\triangleq$ Woman $\sqcap \exists$has-child.Parent
- Mother-without-daughter $\triangleq$ Mother $\sqcap \forall$has-child.Male
- Mother-with-many-children $\triangleq$ Mother $\sqcap (\geq 3 \text{has-child})$

DIANA: Woman
ELIZABETH: Woman
CHARLES: Man
EDWARD: Man
ANDREW: Man
DIANA: Mother-without-daughter
(ELIZABETH, CHARLES): has-child
(ELIZABETH, EDWARD): has-child
(ELIZABETH, ANDREW): has-child
(DIANA, WILLIAM): has-child
(CARL, WILLIAM): has-child

Motivation: Reasoning services

What do we want to know?

We want to check whether the knowledge base is reasonable:
- Is each defined concept in a TBox satisfiable?
- Is a given TBox satisfiable?
- Is a given ABox satisfiable?

What can we conclude from the represented knowledge?
- Is concept $X$ subsumed by concept $Y$?
- Is an object an instance of a concept $X$?

These problems can be reduced to logical satisfiability or implication – using the logical semantics.

However, we take a different route: we will try to simplify these problems and then we specify direct inference methods.
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Basic Reasoning Services
Satisfiability of concept descriptions

Given a concept description $C$ in “isolation”, i.e., in an empty TBox, is $C$ satisfiable?
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- Translated into FOL: Is the formula $\exists x \ C(x)$ satisfiable?
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Example

$\text{Woman} \sqcap (\leq 0 \text{ has-child}) \sqcap (\geq 1 \text{ has-child})$ is unsatisfiable.
Satisfiability of concept descriptions in a TBox

Given a TBox $\mathcal{T}$ and a concept description $C$, is $C$ satisfiable?
Satisfiability of concept descriptions in a TBox

Given a TBox $\mathcal{T}$ and a concept description $C$, is $C$ satisfiable?

Test:

- Does there exist a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^\mathcal{I} \neq \emptyset$?
- Translated into FOL: Is the formula $\exists x \ C(x)$ together with the formulae resulting from the translation of $\mathcal{T}$ satisfiable?
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**Example**

Mother-without-daughter $\sqcap \forall \text{has-child}\text{.Female}$ is unsatisfiable, given our previously specified family TBox.
Reduction: Getting rid of the TBox

We can **reduce** satisfiability problem of concept descriptions in a TBox to the satisfiability problem of concept descriptions in the empty TBox.
Reduction: Getting rid of the TBox

We can reduce satisfiability problem of concept descriptions in a TBox to the satisfiability problem of concept descriptions in the empty TBox.

Idea:

- Since TBoxes are cycle-free, one can understand a concept definition as a kind of “macro”.
- For a given TBox $T$ and a given concept description $C$, all defined concept symbols appearing in $C$ can be expanded until $C$ contains only undefined concept symbols.
- An expanded concept description is then satisfiable if and only if $C$ is satisfiable in $T$.
- **Problem**: What do we do with partial definitions (using $\sqsubseteq$)?
Normalized terminologies

- A terminology is called **normalized** when it does not contain definitions of the form $A \sqsubseteq C$.
- In order to normalize a terminology, replace
  \[ A \sqsubseteq C \]
  by
  \[ A \models A^* \sqcap C, \]
  where $A^*$ is a **fresh** concept symbol (not appearing elsewhere in $\mathcal{T}$).
- If $\mathcal{T}$ is a terminology, the normalized terminology is denoted by $\tilde{\mathcal{T}}$. 

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**Normalizing is reasonable**

**Theorem (Normalization invariance)**

If $\mathcal{I}$ is a model of the terminology $\mathcal{T}$, then there exists a model $\mathcal{I}'$ of $\tilde{\mathcal{T}}$ such that for all concept symbols $A$ occurring in $\mathcal{T}$, it holds $A^{\mathcal{I}} = A^{\mathcal{I}'}$, and *vice versa*.

**Proof.**

"⇒": Let $\mathcal{I}$ be a model of $\mathcal{T}$. This model should be extended to $\mathcal{I}'$ so that the freshly introduced concept symbols also get interpretations.
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**Proof.**

"⇒": Let \( \mathcal{I} \) be a model of \( \mathcal{T} \). This model should be extended to \( \mathcal{I}' \) so that the freshly introduced concept symbols also get interpretations. Assume \( (A \sqsubseteq C) \in \mathcal{T} \), i.e., we have \( (A = A^* \cap C) \in \tilde{\mathcal{T}} \).
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If \( I \) is a model of the terminology \( \mathcal{T} \), then there exists a model \( I' \) of \( \tilde{T} \) such that for all concept symbols \( A \) occurring in \( \mathcal{T} \), it holds \( A^I = A^{I'} \), and vice versa.

Proof.

“\( \Rightarrow \)”: Let \( I \) be a model of \( \mathcal{T} \). This model should be extended to \( I' \) so that the freshly introduced concept symbols also get interpretations. Assume \( (A \sqsubseteq C) \in \mathcal{T} \), i.e., we have \( (A \sqsubseteq A^* \sqcap C) \in \tilde{T} \).

Then set \( A^{*I'} := A^I \).

\( I' \) obviously satisfies \( \tilde{T} \) and has the same interpretation for all symbols in \( \mathcal{T} \).

“\( \Leftarrow \)”: Given a model \( I' \) of \( \tilde{T} \), its restriction to symbols of \( \mathcal{T} \) is the interpretation we look for.
We say that a normalized TBox is unfolded by one step when all defined concept symbols on the right sides are replaced by their defining terms.

Example: Mother $\triangleq$ Woman $\sqcap \ldots$ is unfolded to Mother $\triangleq (\text{Human} \sqcap \text{Female}) \sqcap \ldots$
TBox unfolding

- We say that a normalized TBox is **unfolded by one step** when all defined concept symbols on the right sides are replaced by their defining terms.

- **Example:** Mother ⊑ Woman ⊓ ... is unfolded to Mother ⊑ (Human ⊓ Female) ⊓ ...

- We write $U(\mathcal{T})$ to denote a one-step unfolding and $U^n(\mathcal{T})$ to denote an $n$-step unfolding.

- We say that $\mathcal{T}$ is **unfolded** if $U(\mathcal{T}) = \mathcal{T}$.

- $U^n(\mathcal{T})$ is called the **unfolding** of $\mathcal{T}$ if $U^n(\mathcal{T}) = U^{n+1}(\mathcal{T})$. If such an unfolding exists, it is denoted by $\hat{\mathcal{T}}$. 
Properties of unfoldings (1): Existence

Theorem (Existence of unfolded terminology)

Each normalized terminology $\mathcal{T}$ can be unfolded, i.e., its unfolding $\hat{\mathcal{T}}$ exists.
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*Each normalized terminology $\mathcal{T}$ can be unfolded, i.e., its unfolding $\hat{\mathcal{T}}$ exists.*

Proof idea.

The main reason is that terminologies have to be *cycle-free*. The proof can be done by induction of the definition depth of concepts.
Properties of unfoldings (2): Equivalence

Theorem (Model equivalence for unfolded terminologies)

\( \mathcal{I} \) is a model of a normalized terminology \( \mathcal{T} \) if and only if it is a model of \( \hat{\mathcal{T}} \).
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**Theorem (Model equivalence for unfolded terminologies)**

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**Proof sketch.**

\(\Rightarrow\): Let \(\mathcal{I}\) be a model of \(\mathcal{T}\).
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\( \Rightarrow \): Let \( \mathcal{I} \) be a model of \( \mathcal{T} \). Then it is also a model of \( U(\mathcal{T}) \), since on the right side of the definitions only terms with identical interpretations are substituted.
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\[ \Leftarrow \]: Let \( \mathcal{I} \) be a model for \( U(\mathcal{T}) \). Clearly, this is also a model of \( \mathcal{T} \) (with the same argument as above).

This means that any model \( \hat{\mathcal{T}} \) is also a model of \( \mathcal{T} \).
Generating models

- All concept and role names not occurring on the left hand side of definitions in a terminology $\mathcal{T}$ are called **primitive components**.
- Interpretations restricted to primitive components are called **initial interpretations**.

**Theorem (Model extension)**

*For each initial interpretation $\mathcal{I}$ of a normalized TBox, there exists a unique interpretation $\mathcal{I}$ extending $\mathcal{J}$ and satisfying $\mathcal{T}$.***
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Use $\hat{T}$ and compute an interpretation for all defined symbols.

**Corollary (Model existence for TBoxes)**

Each TBox has at least one model.
Unfolding of concept descriptions

- Similar to the unfolding of TBoxes, we can define the unfolding of a concept description.
- We write $\hat{C}$ for the unfolded version of $C$.

**Theorem (Satisfiability of unfolded concepts)**

An concept description $C$ is satisfiable in a terminology $T$ if and only if $\hat{C}$ satisfiable in an empty terminology.
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Then extend it to a full model \( I \) of \( T \).
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$\Leftarrow$: Use the interpretation for all the symbols in $\hat{C}$ to generate an initial interpretation of $\mathcal{T}$.
Then extend it to a full model $\mathcal{I}$ of $\mathcal{T}$.
This satisfies $\mathcal{T}$ as well as $\hat{C}$. Since $\hat{C}^\mathcal{I} = C^\mathcal{I}$, it satisfies also $C$. \qed
General TBox Reasoning Services
Subsumption in a TBox

Given a terminology $T$ and two concept descriptions $C$ and $D$, is $C$ subsumed by (or a sub-concept of) $D$ in $T$ (symb. $C \sqsubseteq_T D$)?

Test:

- Is $C$ interpreted as a subset of $D$ in each model $\mathcal{I}$ of $T$, i.e. $C^\mathcal{I} \subseteq D^\mathcal{I}$?
- Is the formula $\forall x (C(x) \rightarrow D(x))$ a logical consequence of the translation of $T$ into FOL?
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Example

Given our family TBox, it holds Grandmother $\sqsubseteq_\mathcal{T}$ Mother.
Subsumption (without a TBox)

Given two concept descriptions $C$ and $D$, is $C$ subsumed by $D$ regardless of a TBox (or in an empty TBox) (symb. $C \sqsubseteq D$)?

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- Is $C$ interpreted as a subset of $D$ for all interpretations $\mathcal{I}$ ($C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$)?
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Example

Clearly, Human $\sqcap$ Female $\sqsubseteq$ Human.
Subsumption in a TBox can be reduced to subsumption in the empty TBox:

… normalize and unfold TBox and concept descriptions.
Reductions

- Subsumption in a TBox can be reduced to subsumption in the empty TBox:
  
  ... normalize and unfold TBox and concept descriptions.

- Subsumption in the empty TBox can be reduced to unsatisfiability:
  
  ... $C \sqsubseteq D$ iff $C \sqcap \neg D$ is unsatisfiable.
Reductions

- Subsumption in a TBox can be reduced to subsumption in the empty TBox:
  ... normalize and unfold TBox and concept descriptions.

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- Unsatisfiability can be reduced to subsumption:
  ... $C$ is unsatisfiable iff $C \sqsubseteq (C \cap \neg C)$. 
Classification

Compute all subsumption relationships (and represent them using only a minimal number of relationships)!
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Useful in order to:

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Problem can be reduced to subsumption checking: then it is a generalized sorting problem!
General ABox Reasoning Services
ABox satisfiability

Satisfiability of an ABox

Given an ABox $\mathcal{A}$, does this set of assertions have a model?

Notice: ABoxes representing the real world should always have a model.

Example: The ABox $\mathcal{X}$: $(\forall r. \neg C)$, $\mathcal{Y}: C$, $(\mathcal{X}, \mathcal{Y}) : r$ is not satisfiable.
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Example

The ABox

$$X : (\forall r. \neg C), \quad Y : C, \quad (X, Y) : r$$

is not satisfiable.
ABox satisfiability in a TBox

Given an ABox $\mathcal{A}$ and a TBox $\mathcal{T}$, is $\mathcal{A}$ consistent with the terminology introduced in $\mathcal{T}$, i.e., is $\mathcal{T} \cup \mathcal{A}$ satisfiable?

Example

If we extend our example with

- MARGRET: Woman
- (DIANA,MARGRET): has-child,

then the ABox becomes unsatisfiable in the given TBox.
ABox satisfiability in a TBox

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then the ABox becomes unsatisfiable in the given TBox.

- Problem is reducible to satisfiability of an ABox:
  - … normalize terminology, then unfold all concept and role descriptions in the ABox
Instance relations

Which additional ABox formulae of the form \( a : C \) follow logically from a given ABox and TBox?

- Is \( a^I \in C^I \) true in all models \( I \) of \( T \cup A \)?
- Does the formula \( C(a) \) logically follow from the translation of \( A \) and \( T \) to predicate logic?
Instance relations

Which additional ABox formulae of the form \( a : C \) follow logically from a given ABox and TBox?

- Is \( a^T \in C^T \) true in all models \( \mathcal{I} \) of \( \mathcal{T} \cup \mathcal{A} \)?
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Reductions:
- Instance relations wrt. an ABox and a TBox can be reduced to instance relations wrt. ABox: use normalization and unfolding
 Instance relations

Which additional ABox formulae of the form $a : C$ follow logically from a given ABox and TBox?

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Reductions:
- Instance relations wrt. an ABox and a TBox can be reduced to instance relations wrt. ABox: use normalization and unfolding
- Instance relations in an ABox can be reduced to ABox unsatisfiability:

  $a : C$ holds in $\mathcal{A} \iff \mathcal{A} \cup \{a : \neg C\}$ is unsatisfiable
Examples

Example

- ELIZABETH: Mother-with-many-children?
Examples

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  yes
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Example

- **ELIZABETH:** Mother-with-many-children?
  - yes
- **WILLIAM:** ¬ Female?
  - yes
  - no (no CWA!)
  - no (only male, but not necessarily human!)
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  yes

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  yes

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  yes
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- ELIZABETH: Grandmother?
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Realization

For a given object \( a \), determine the most specialized concept symbols such that \( a \) is an instance of these concepts.

Motivation:
- Similar to classification
- Is the minimal representation of the instance relations (in the set of concept symbols)
- Will give us faster answers for instance queries!
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**Motivation:**

- Similar to classification
- Is the minimal representation of the instance relations (in the set of concept symbols)
- Will give us faster answers for instance queries!

**Reduction:** Can be reduced to (a sequence of) instance relation tests.
Given a concept description $C$, determine the set of all (specified) instances of the concept description.

We ask for all instances of the concept Male. For our TBOX/ABox we will get the answer CHARLES, ANDREW, EDWARD, WILLIAM.
Retrieval

Given a concept description $C$, determine the set of all (specified) instances of the concept description.

Example

We ask for all instances of the concept Male.
For our TBOX/ABox we will get the answer CHARLES, ANDREW, EDWARD, WILLIAM.

- **Reduction**: Compute the set of instances by testing the instance relation for each object!
- **Implementation**: Realization can be used to speed this up
Summary and Outlook
Reasoning services – summary

- Satisfiability of concept descriptions
  - in a given TBox or in an empty TBox
- Subsumption between concept descriptions
  - in a given TBox or in an empty TBox
- Classification
- Satisfiability of an ABox
  - in a given TBox or in an empty TBox
- Instance relations in an ABox
  - in a given TBox or in an empty TBox
- Realization
- Retrieval
Outlook

- How to determine subsumption between two concept descriptions (in the empty TBox)?
- How to determine instance relations/ABox satisfiability?
- How to implement the mentioned reductions efficiently?
- Does normalization and unfolding introduce another source of computational complexity?