Principles of Knowledge Representation and Reasoning

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Exercise Sheet 11

Due: January 20th, 2016

Exercise 11.1 (Cumulative logics, 3)

Show that in system **C** the rule (MPC) can be derived.

$$\frac{\alpha \mid\!\sim \beta \to \gamma, \quad \alpha \mid\!\sim \beta}{\alpha \mid\!\sim \gamma}$$

In your proof you may use the five basic rules of system **C**, the derived rules *Supraclassicality*, *Equivalence*, and *And*, as well as propositional logic tautologies.

Exercise 11.2 (Cumulative models, 3)

We consider the rules *EHD*, *Cumulativity*, and *Monotonicity*. Which of these rules are valid in all cumulative models? Provide proofs or counterexamples.

Exercise 11.3 (Cumulative Logics and Probabilities, 5)

In what follows we consider probability functions p on the set of all propositional logic formulae over some fixed finite alphabet. That is, p is a finitely additive probability function (satisfying the usual Kolmogorov axioms) that assigns to each formula its probability in the real interval $[0, 1]$.

Given such a probability function p and a threshold $\vartheta \in [0, 1]$, we define the *probabilistic consequence relation* $\mid\!\sim_{p,\vartheta}$ as follows:

$$\alpha \mid\!\sim_{p,\vartheta} \beta \iff p(\alpha) = 0 \text{ or } p(\beta|\alpha) \geq \vartheta$$

—here $p(\beta|\alpha)$ denotes the conditional probability. For a conditional $\alpha \mid\!\sim \beta$, we say that the pair p, ϑ *satisfies* the conditional if and only if $\alpha \mid\!\sim_{p,\vartheta} \beta$.

A cumulative rule is called *probabilistically sound* if each probabilistic consequence relation is closed under the rule, that is, if each choice of p, ϑ that satisfies each conditional in the premise of the rule, also satisfies the conditional in the conclusion of the rule.

Which of the following rules are probabilistically sound, and which are not: Supra-Classicality, Right Weakening, Cautious Monotonicity, And, Proof-By-Case-Analysis (D)? Provide proofs or counter-examples.