Principles of Knowledge Representation and Reasoning

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Winter Semester 2015/2016

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Exercise Sheet 10

Due: January 13, 2015

Exercise 10.1 (Properties of Default Logic, 3 + 3 + 3)

Proof or disproof:

(a) Let $\langle D, W\rangle$ be a propositional default theory and let D' be a set of normal defaults with $D \subseteq D'$. If E is an extension of $\langle D, W\rangle$, then there exists an extension E' of $\langle D', W\rangle$ such that $E \subseteq E'$.

(b) Let $\langle D, W\rangle$ be a propositional default theory and ϕ be a formula that skeptically follows from $\langle D, W\rangle$. Then, each formula that skeptically follows from $\langle D, W \cup \{\phi\}\rangle$ also skeptically follows from $\langle D, W\rangle$, and vice versa.

(c) Let $\langle D, W\rangle$ be a propositional, semi-normal default theory that has a consistent extension E such that the justification (consistency condition) of each default rule in D is consistent with E (und thus with W). Then W must be consistent with the set of all justifications of the default rules in D.

Exercise 10.2 (Fixpoint semantics for default logic, 3)

Let $\langle D, W\rangle$ be a propositional default theory. For a given set of formulae, X, we consider sets of formulae U with the following properties:

(i) $W \subseteq U$,

(ii) $\mathrm{Th}(U) = U$,

(iii) If $\frac{\alpha : \beta}{\gamma} \in D$, $\alpha \in U$, and $X \cup \{\beta\}$ is consistent, then $\gamma \in U$.

Let Γ be the operator that assigns to each set X of formulae the smallest set of formulae, $\Gamma(X)$, that has these properties (Γ is well-defined because for each set of formulae X such a smallest set $\Gamma(X)$ exists).

Show that a set E of formulae is an extension of $\langle D, W\rangle$ if and only if E is a fixpoint of Γ (i.e., $\Gamma(E) = E$).

Hint: For an arbitrary set of formulae E consider the sets $E_0 := W$, $E_i := \mathrm{Th}(E_{i-1}) \cup \dots$ (as defined in the lecture). Show first that $\bigcup_i E_i$ satisfies the conditions (i)–(iii).