Principles of Knowledge Representation and Reasoning

B. Nebel, S. Wölfl, F. Lindner
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University of Freiburg
Department of Computer Science

Exercise Sheet 8

Due: December 16th, 2015

Exercise 8.1 (Tableau Algorithm and Reasoning Services, 2 + 2 + 2)

In this exercise you are asked to apply the tableau algorithm for description logics (lecture 9) and to use it to answer questions about TBoxes and ABoxes.[1]

(a) Use the tableau algorithm to show that the $\mathcal{ALC}$ concept $C \doteq \forall r.(\neg A \sqcup \exists s.A) \sqcap \exists r.(A \sqcap \exists s.\neg A)$ is satisfiable. Extract a model of C from your tableau.

(b) Given two $\mathcal{ALCQ}$ concepts A and B, use tableau to prove that $A \sqsubseteq B$.
Hint: To transform a concept description that contains (qualified) number restrictions to negation normal form, you may need to make use of the following equivalences: $\neg(\geq n+1r.C) \equiv (\leq nr.C), \neg(\geq 0r.C) \equiv \bot, \neg(\leq nr.C) \equiv (\geq n+1r.C)$. The rule for expanding (qualified) number restrictions is explained in the Figure 2.6 mentioned in the footnote. Moreover, one needs to extend the clash detection, viz., there is a clash in a branch $\mathcal{B}$ if $\{(\leq nR(x))\} \cup \{R(x, y_i) | 1 \leq i \leq n+1\} \cup \{y_i \neq y_j | 1 \leq i < j \leq n+1\} \subseteq \mathcal{B}$ for individual names $x, y_1, \ldots, y_{n+1}$, a nonnegative integer n, and a role name R.[2].

- $A \doteq \exists r.(\leq 2r.C) \sqcap \forall r.C$
- $B \doteq \forall r.(C \sqcup D) \sqcap \exists r.(\leq 3r.C)$

(c) Explain how one can you use the tableau procedure to retrieve all instances of the concept C' given the ABox $\mathcal{A}$. Exemplify your approach by choosing any of the individuals and prove or disprove that it is an instance of C'.

- $\mathcal{A} = \{A(a), A(c), B(b), \neg C(d), r(b,c), r(c,d), s(a,b), s(a,c), s(c,c)\}$
- $C' \doteq \exists s.B \sqcup \exists s.\exists r.\neg C$

Exercise 8.2 (Expressibility and Complexity, 3 + 3)

(a) You are asked to show that concept constructors are not just syntactic sugar. In particular, if one adds the functionality concept constructor — $(\leq 1r)$ — to $\mathcal{ALC}$ one gets the language $\mathcal{ALCF}$. Show that $\mathcal{ALCF}$ is indeed more expressive than $\mathcal{ALC}$ by proving that the concept $(\leq 1r)$ cannot be expressed in $\mathcal{ALC}$.
Hint: Assume an $\mathcal{ALC}$ concept C being equivalent to $(\leq 1r)$. Provide an interpretation $\mathcal{I}$ which satisfies both C and $(\leq 1r)$. Then construct another interpretation $\mathcal{I}'$ from $\mathcal{I}$ as follows: $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}} \times \mathbb{N}, A^{\mathcal{I}'} = \{(d,i) | d \in A^{\mathcal{I}}, i \in \mathbb{N})\}, r^{\mathcal{I}'} = \{(< d, i >, < e, j >) | (d,e) \in r^{\mathcal{I}}, i, j \in \mathbb{N}\}$ and show that $\mathcal{I}'$ is a model for C but not for $(\leq 1r)$.

[1] You may also want to consult `https://www.inf.unibz.it/~franconi/dl/course/dlhb/dlhb-02.pdf` (p. 85, Fig. 2.6) for an overview of some basic tableau expansion rules.

[2] Cf., `https://www.inf.unibz.it/~franconi/dl/course/dlhb/dlhb-02.pdf`, page 86

(b) Given the TBox $\mathcal{T}$, determine the description logic used to describe the concepts. What is the complexity class of the satisfiability problem of this logic? Propose how $\mathcal{T}$ could be expressed in a less complex DL.

- $\mathcal{T} = \{A \doteq \exists r.(\forall s.C \sqcup \exists s.\neg C), B \doteq (\geq 1 s), C \doteq B \sqcap \forall r.\neg(\neg A \sqcup \neg B)\}$