Exercise 11.1 (Strong stubborn sets, 1+3 points)

Consider the SAS+ planning task $\Pi$ with variables $V = \{pos, left, right, hat\}$, $D_{pos} = \{home, uni\}$ and $D_{left} = D_{right} = D_{hat} = \{t, f\}$. The initial state is $I = \{pos \leftrightarrow home, left \leftrightarrow f, right \leftrightarrow f, hat \leftrightarrow f\}$ and the goal specification is $\gamma = \{pos \leftrightarrow uni\}$. There are four operators in $O$, namely

- $wear-left-shoe = \langle pos = home \land left = f, left := t \rangle$,
- $wear-right-shoe = \langle pos = home \land right = f, right := t \rangle$,
- $wear-hat = \langle pos = home \land hat = f, hat := t \rangle$, and
- $go-to-university = \langle pos = home \land left = t \land right = t, pos := uni \rangle$.

(a) Draw the breadth-first search graph (with duplicate detection) for planning task $\Pi$ without any form of partial-order reduction.

(b) Draw the breadth-first search graph (with duplicate detection) for planning task $\Pi$ using strong stubborn set pruning. For each expansion of a node for a state $s$, specify in detail how $T_s$ (and thus $T_{app(s)}$) are computed, i.e., explain how the initial disjunctive action landmark is chosen and how operators are iteratively added to $T_s$ as part of necessary enabling sets or interfering operators, respectively. Break ties in favor of $wear-left-shoe$ over $wear-right-shoe$.

How many node expansion do you save with strong stubborn sets compared to plain breadth-first search? What about the lengths of the resulting solutions?

You may abbreviate the operator names, state descriptions etc. appropriately.

Exercise 11.2 (Weak vs. strong stubborn sets, 6 points)

Show that weak stubborn sets admit exponentially more pruning than strong stubborn sets.

Hint: Consider the family of planning tasks $(\Pi_n)_{n \in \mathbb{N}}$, where $\Pi_n = (V_n, I_n, O_n, \gamma)$ is the planning task with the following components:

- $V_n = \{a, x, y, b_1, \ldots, b_n\}$ with variable domains $D_a = D_x = D_y = \{0, 1\}$ and $D_{b_i} = \{0, 1, 2\}$ for all $i \in \{1, \ldots, n\}$
- $O_n = \{o, a, o_1, \ldots, o_n, \overline{a}, \overline{o_1}, \ldots, \overline{o_n}\}$
- $pre(a) = \{a \rightarrow 0\}, eff(a) = \{x \rightarrow 1\}$
- $pre(a') = \{a \rightarrow 0\}, eff(a') = \{y \rightarrow 1\}$
- $pre(\overline{a}) = \{a \rightarrow 0\}, eff(\overline{a}) = \{a \rightarrow 1, b_1 \rightarrow 1, \ldots, b_n \rightarrow 1\}$
- $pre(\overline{a}) = \{a \rightarrow 1\}, eff(\overline{a}) = \{a \rightarrow 0, b_1 \rightarrow 1, \ldots, b_n \rightarrow 1\}$
- $pre(o_i) = \{b_i \rightarrow 1\}, eff(o_i) = \{b_i \rightarrow 2\}$ for $1 \leq i \leq n$
- $pre(\overline{o_i}) = \{b_i \rightarrow 2\}, eff(\overline{o_i}) = \{b_i \rightarrow 1\}$ for $1 \leq i \leq n$
- $I_n = \{a \rightarrow 0, x \rightarrow 0, y \rightarrow 0, b_1 \rightarrow 0, \ldots, b_n \rightarrow 0\}$
- $\gamma = \{x \rightarrow 1, y \rightarrow 1\}$

You can and should solve the exercise sheets in groups of two. Please state both names on your solution.