Exercise 5.1 (Domination lemma, 3 points)
Let \( s, s' : A \rightarrow \{0,1\} \) be valuations for a set \( A \) of state variables and let \( \chi \) be a negation-free formula over \( A \). Show by structural induction on \( \chi \): If \( s \models \chi \) and \( s' \) dominates \( s \), then \( s' \models \chi \).

Exercise 5.2 (Delete relaxation, 1+2 points)
Consider the planning task \( \Pi = \langle A, I, O, \gamma \rangle \) in positive normal form with
\[
A = \{ \text{haveCake}, \text{eatenCake}, \text{haveNoCake} \},
\]
\[
I = \{ \text{have-cake} \mapsto 0, \text{eatenCake} \mapsto 0, \text{haveNoCake} \mapsto 1 \}
\]
\[
O = \{ \text{eatCake}, \text{bakeCake} \},
\]
\[
eatCake = \langle \text{have-cake} \mapsto \neg \text{haveCake} \land \text{haveNoCake} \land \text{eatenCake} \rangle,
\]
\[
bakeCake = \langle \text{haveNoCake}, \text{haveCake} \land \neg \text{haveNoCake} \rangle \text{ und}
\]
\[
\gamma = \text{haveCake} \land \text{eatenCake}.
\]

(a) Give the relaxation \( \Pi^+ \) of \( \Pi \).
(b) Give a sequence \( \pi \) of operators (as short as possible) from \( O \) such that \( \pi \) is not a plan of \( \Pi \), but \( \pi^+ \) is a plan of \( \Pi^+ \).

Exercise 5.3 (h+ heuristic, 2+2 points)
A 15-puzzle planning task \( \Pi = \langle A, I, O, \gamma \rangle \) is given as
\[
A = \{ \text{empty}(p_{i,j}) \mid 0 \leq i, j \leq 3 \} \cup \{ \text{at}(t_k, p_{i,j}) \mid 0 \leq i, j \leq 3, 0 \leq k \leq 14 \},
\]
\[
O = \{ \text{move}(t_m, p_{i,j}, p_{k,l}) \mid 0 \leq i, j, k, l \leq 3, 0 \leq m \leq 14, (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1) \},
\]
\[
\gamma = \bigwedge_{0 \leq m \leq 14} \text{at}(t_m, p_{m/4 \mod 4, m/4})
\]
Action \( \text{move}(t_m, p_{i,j}, p_{k,l}) \) moves tile \( t_m \) from position \( p_{i,j} \) to position \( p_{k,l} \):
\[
\text{move}(t_m, p_{i,j}, p_{k,l}) = (\text{at}(t_m, p_{i,j}) \land \text{empty}(p_{k,l})),
\]
\[
\text{at}(t_m, p_{k,l}) \land \text{empty}(p_{i,j}) \land \neg \text{at}(t_m, p_{i,j}) \land \neg \text{empty}(p_{k,l})
\]
A syntactically possible state is legal if each tile \( t_m \) is at some position \( p_{i,j} \), if no two tiles are at the same position and if the remaining position is the only one that is empty. The initial state is an arbitrary state that is legal.

One possible heuristic for the 15-puzzle is the Manhattan-distance heuristic \( h^{\text{Manhattan}} \): It sums the Manhattan distances of all tiles from their current positions to their target positions, where the Manhattan distance between position \( p_{i,j} \) and \( p_{k,l} \) is given as \( |i - k| + |j - l| \).
The \( h^+ \) heuristic estimates the distance of state \( s \) to the closest goal state as the length of the optimal plan in the relaxed planning task (with initial state \( s \)).

(a) Show that \( h^+(s) \geq h^{\text{Manhattan}}(s) \) for each legal state \( s \) of a 15-puzzle planning task.
(b) Show that \( h^+(s) > h^{\text{Manhattan}}(s) \) for at least one state \( s \) of a 15-puzzle planning task.

You can and should solve the exercise sheets in groups of two. Please state both names on your solution.