Principles of AI Planning

5. Planning as search: progression and regression

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October 31st, 2014
1 Planning as (classical) search

- Introduction
- Classification of search-based planners
What do we mean by search?

- **Search** is a very generic term.
- Every algorithm that tries out various alternatives can be said to “search” in some way.
- Here, we mean **classical search** algorithms.
  - **Search nodes** are expanded to generate successor nodes.
  - **Examples:** breadth-first search, A*, hill-climbing, …
- To be brief, we just say **search** in the following (not “classical search”).
Do you know this stuff already?

- **We assume prior knowledge** of basic search algorithms:
  - uninformed vs. informed
  - systematic vs. local
- There will be a small refresher in the next chapter.
- **Background:** Russell & Norvig, Artificial Intelligence – A Modern Approach, Ch. 3 (all of it), Ch. 4 (local search)
Search in planning

- search: one of the big success stories of AI
- many planning algorithms based on classical AI search (we’ll see some other algorithms later, though)
- will be the focus of this and the following chapters (the majority of the course)
Satisficing or optimal planning?

Must carefully distinguish two different problems:

- **satisficing planning**: any solution is OK (although shorter solutions typically preferred)
- **optimal planning**: plans must have shortest possible length

Both are often solved by search, but:

- details are very different
- almost no overlap between good techniques for satisficing planning and good techniques for optimal planning
- many problems that are trivial for satisficing planners are impossibly hard for optimal planners
Planning by search

How to apply search to planning? \(\sim\) many choices to make!

Choice 1: Search direction

- progression: forward from initial state to goal
- regression: backward from goal states to initial state
- bidirectional search
Planning by search

How to apply search to planning? \(\leadsto\) many choices to make!

Choice 2: Search space representation

- search nodes are associated with states
  \(\leadsto\) state-space search
- search nodes are associated with sets of states
Planning by search

How to apply search to planning? ⇒ many choices to make!

Choice 3: Search algorithm

- uninformed search:
  depth-first, breadth-first, iterative depth-first, ...

- heuristic search (systematic):
  greedy best-first, A*, Weighted A*, IDA*, ...

- heuristic search (local):
  hill-climbing, simulated annealing, beam search, ...
Planning by search

How to apply search to planning? → many choices to make!

Choice 4: Search control

- heuristics for informed search algorithms
- pruning techniques: invariants, symmetry elimination, partial-order reduction, helpful actions pruning, …
Search-based satisficing planners

FF (Hoffmann & Nebel, 2001)
- search direction: forward search
- search space representation: single states
- search algorithm: enforced hill-climbing (informed local)
- heuristic: FF heuristic (inadmissible)
- pruning technique: helpful actions (incomplete)

⇝ one of the best satisficing planners
Search-based optimal planners

Fast Downward Stone Soup (Helmert et al., 2011)

- **search direction**: forward search
- **search space representation**: single states
- **search algorithm**: $A^*$ (informed systematic)
- **heuristic**: multiple admissible heuristics combined into a heuristic portfolio (LM-cut, M&S, blind, …)
- **pruning technique**: none

⇝ one of the best optimal planners
Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   ⇝ this chapter

2. search space representation: states/sets of states
   ⇝ this chapter

3. search algorithm: uninformed/heuristic; systematic/local
   ⇝ next chapter

4. search control: heuristics, pruning techniques
   ⇝ following chapters
2 Progression

- Overview
- Example
Planning by forward search: progression

**Progression**: Computing the successor state $app_o(s)$ of a state $s$ with respect to an operator $o$.

**Progression planners** find solutions by forward search:

- start from initial state
- iteratively pick a previously generated state and **progress** it through an operator, generating a new state
- solution found when a goal state generated

**pro**: very easy and efficient to implement
Two alternative search spaces for progression planners:

1. **search nodes correspond to states**
   - when the same state is generated along different paths, it is not considered again (duplicate detection)
   - **pro:** save time to consider same state again
   - **con:** memory intensive (must maintain closed list)

2. **search nodes correspond to operator sequences**
   - different operator sequences may lead to identical states (transpositions); search does not notice this
   - **pro:** can be very memory-efficient
   - **con:** much wasted work (often exponentially slower)

⇒ first alternative usually preferable in planning
(Unlike many classical search benchmarks like 15-puzzle)
Example where search nodes correspond to operator sequences (no duplicate detection)
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(no duplicate detection)
Example where search nodes correspond to operator sequences (no duplicate detection)
Progression planning example (depth-first search)

Example where search nodes correspond to operator sequences (no duplicate detection)
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Progression planning example (depth-first search)

**Example** where search nodes correspond to operator sequences (no duplicate detection)
Example where search nodes correspond to operator sequences (no duplicate detection)
3 Regression

- Overview
- Example
- Regression for STRIPS tasks
- Regression for general planning tasks
- Practical issues
Going through a transition graph in forward and backward directions is not symmetric:

- forward search starts from a single initial state; backward search starts from a set of goal states
- when applying an operator $o$ in a state $s$ in forward direction, there is a unique successor state $s'$;
  if we applied operator $o$ to end up in state $s'$, there can be several possible predecessor states $s$

→ most natural representation for backward search in planning associates sets of states with search nodes
Planning by backward search: regression

Regression: Computing the possible predecessor states $\text{regr}_o(G)$ of a set of states $G$ with respect to the last operator $o$ that was applied.

Regression planners find solutions by backward search:
- start from set of goal states
- iteratively pick a previously generated state set and regress it through an operator, generating a new state set
- solution found when a generated state set includes the initial state

Pro: can handle many states simultaneously
Con: basic operations complicated and expensive
identify state sets with logical formulae (again):

- search nodes correspond to state sets
- each state set is represented by a logical formula:
  \[ \varphi \text{ represents } \{ s \in S \mid s \models \varphi \} \]
- many basic search operations like detecting duplicates are NP-hard or coNP-hard
Regression planning example (depth-first search)

\[
\begin{align*}
\phi_1 & = \text{regr} \rightarrow (\gamma) \\
\phi_2 & = \text{regr} \rightarrow (\phi_1) \\
\phi_3 & = \text{regr} \rightarrow (\phi_2), \\
I | & = \phi_3
\end{align*}
\]
Regression planning example (depth-first search)

\[ \phi_1 = \text{regr} \rightarrow (\gamma) \]

\[ \phi_2 = \text{regr} \rightarrow (\phi_1) \]

\[ \phi_3 = \text{regr} \rightarrow (\phi_2) \]

\[ I | \phi = \phi_3 \]

October 31st, 2014
Regression planning example (depth-first search)

\[ \varphi_1 = \text{regr} \rightarrow (\gamma) \]

\[ \varphi_1 \rightarrow \gamma \]
Regression planning example (depth-first search)

\[ \varphi_1 = \text{regr}(\gamma) \]
\[ \varphi_2 = \text{regr}(\varphi_1) \]

\[ \varphi_2 \rightarrow \varphi_1 \rightarrow \gamma \]
Regression planning example (depth-first search)

\[ \varphi_1 = \text{regr}(\gamma) \]
\[ \varphi_2 = \text{regr}(\varphi_1) \]
\[ \varphi_3 = \text{regr}(\varphi_2), \models \varphi_3 \]
Regression for STRIPS planning tasks

Definition (STRIPS planning task)

A planning task is a STRIPS planning task if all operators are STRIPS operators and the goal is a conjunction of atoms.

Regression for STRIPS planning tasks is very simple:

- Goals are conjunctions of atoms $a_1 \land \cdots \land a_n$.
- First step: Choose an operator that makes none of $a_1, \ldots, a_n$ false.
- Second step: Remove goal atoms achieved by the operator (if any) and add its preconditions.

Outcome of regression is again conjunction of atoms.

Optimization: only consider operators making some $a_i$ true.
Definition (STRIPS regression)

Let $\varphi = \varphi_1 \land \cdots \land \varphi_n$ be a conjunction of atoms, and let $o = \langle \chi, e \rangle$ be a STRIPS operator which adds the atoms $a_1, \ldots, a_k$ and deletes the atoms $d_1, \ldots, d_l$.

The STRIPS regression of $\varphi$ with respect to $o$ is

$$sregr_o(\varphi) := \begin{cases} 
\bot & \text{if } a_i = d_j \text{ for some } i, j \\
\bot & \text{if } \varphi_i = d_j \text{ for some } i, j \\
\chi \land \bigwedge (\{\varphi_1, \ldots, \varphi_n\} \setminus \{a_1, \ldots, a_k\}) & \text{otherwise}
\end{cases}$$

Note: $sregr_o(\varphi)$ is again a conjunction of atoms, or $\bot$. 
STRIPS regression example

Note: Predecessor states are in general not unique. This picture is just for illustration purposes.

\[
\begin{align*}
o_1 &= \langle \text{on} \land \text{clr}, \neg \text{on} \land \text{onT} \land \text{clr} \rangle \\
o_2 &= \langle \text{on} \land \text{clr} \land \text{clr}, \neg \text{clr} \land \neg \text{on} \land \text{on} \land \text{clr} \rangle \\
o_3 &= \langle \text{onT} \land \text{clr} \land \text{clr}, \neg \text{clr} \land \neg \text{onT} \land \text{on} \rangle \\
\gamma &= \text{on} \land \text{on} \\
\varphi_1 &= \text{sregr}_{o_3}(\gamma) = \text{onT} \land \text{clr} \land \text{clr} \land \text{on} \\
\varphi_2 &= \text{sregr}_{o_2}(\varphi_1) = \text{on} \land \text{clr} \land \text{clr} \land \text{onT} \\
\varphi_3 &= \text{sregr}_{o_1}(\varphi_2) = \text{on} \land \text{clr} \land \text{on} \land \text{onT}
\end{align*}
\]
Regression for general planning tasks

- With disjunctions and conditional effects, things become more tricky. How to regress \( a \lor (b \land c) \) with respect to \( \langle q, d \triangleright b \rangle \)?

- The story about goals and subgoals and fulfilling subgoals, as in the STRIPS case, is no longer useful.

- We present a general method for doing regression for any formula and any operator.

- Now we extensively use the idea of representing sets of states as formulae.
Effect preconditions

Definition (effect precondition)

The effect precondition $EPC_l(e)$ for literal $l$ and effect $e$ is defined as follows:

\[
EPC_l(l) = \top \\
EPC_l(l') = \bot \text{ if } l \neq l' \text{ (for literals } l') \\
EPC_l(e_1 \land \cdots \land e_n) = EPC_l(e_1) \lor \cdots \lor EPC_l(e_n) \\
EPC_l(\chi \triangleright e) = EPC_l(e) \land \chi
\]

Intuition: $EPC_l(e)$ describes the situations in which effect $e$ causes literal $l$ to become true.
Effect precondition examples

Example

\[ EPC_a(b \land c) = \bot \lor \bot \equiv \bot \]
\[ EPC_a(a \land (b \triangleright a)) = \top \lor (\top \land b) \equiv \top \]
\[ EPC_a((c \triangleright a) \land (b \triangleright a)) = (\top \land c) \lor (\top \land b) \equiv c \lor b \]
Effect preconditions: connection to change sets

Lemma (A)

Let \( s \) be a state, \( l \) a literal and \( e \) an effect. Then \( l \in [e]_s \) if and only if \( s \models EPC_i(e) \).

Proof.

Induction on the structure of the effect \( e \).
Base case 1, \( e = l \): \( l \in [l]_s = \{l\} \) by definition, and \( s \models EPC_i(l) = \top \) by definition. Both sides of the equivalence are true.
Base case 2, \( e = l' \) for some literal \( l' \neq l \): \( l \notin [l']_s = \{l'\} \) by definition, and \( s \not\models EPC_i(l') = \bot \) by definition. Both sides are false.
Effect preconditions: connection to change sets

Proof (ctd.)

Inductive case 1, \( e = e_1 \land \cdots \land e_n \):
\[
\begin{align*}
 l \in [e]_s & \iff l \in [e_1]_s \cup \cdots \cup [e_n]_s \\
 & \quad \text{(Def } [e_1 \land \cdots \land e_n]_s) \\
 & \iff l \in [e']_s \text{ for some } e' \in \{e_1, \ldots, e_n\} \\
 & \quad \text{iff } s \models EPC_i(e') \text{ for some } e' \in \{e_1, \ldots, e_n\} \quad \text{(IH)} \\
 & \quad \text{iff } s \models EPC_i(e_1) \lor \cdots \lor EPC_i(e_n) \\
 & \quad \text{iff } s \models EPC_i(e_1 \land \cdots \land e_n). \quad \text{(Def } EPC) 
\end{align*}
\]

Inductive case 2, \( e = \chi \triangleright e' \):
\[
\begin{align*}
 l \in [\chi \triangleright e']_s & \iff l \in [e']_s \text{ and } s \models \chi \\
 & \quad \text{(Def } [\chi \triangleright e']_s) \\
 & \iff s \models EPC_i(e') \text{ and } s \models \chi \quad \text{(IH)} \\
 & \quad \text{iff } s \models EPC_i(e') \land \chi \\
 & \quad \text{iff } s \models EPC_i(\chi \triangleright e'). \quad \text{(Def } EPC) 
\end{align*}
\]
Remark: *EPC* vs. effect normal form

Notice that in terms of *EPC*\(_a(e)\), any operator \(\langle \chi, e \rangle\) can be expressed in effect normal form as

\[
\left\langle \chi, \bigwedge_{a \in A} ((EPC_a(e) \triangleright a) \land (EPC_{\neg a}(e) \triangleright \neg a)) \right\rangle,
\]

where \(A\) is the set of all state variables.
Regressing state variables

The formula $EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$ expresses the value of state variable $a \in A$ after applying $o$ in terms of values of state variables before applying $o$.

Either:

- $a$ became true, or
- $a$ was true before and it did not become false.
Regressing state variables: examples

**Example**

Let $e = (b \triangleright a) \land (c \triangleright \neg a) \land b \land \neg d$.

<table>
<thead>
<tr>
<th>variable $x$</th>
<th>$EPC_x(e) \lor (x \land \neg EPC_{\neg x}(e))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$b \lor (a \land \neg c)$</td>
</tr>
<tr>
<td>$b$</td>
<td>$\top \lor (b \land \neg \bot) \equiv \top$</td>
</tr>
<tr>
<td>$c$</td>
<td>$\bot \lor (c \land \neg \bot) \equiv c$</td>
</tr>
<tr>
<td>$d$</td>
<td>$\bot \lor (d \land \neg \top) \equiv \bot$</td>
</tr>
</tbody>
</table>
Lemma (B)

Let \( a \) be a state variable, \( o = \langle \chi, e \rangle \) an operator, 
\( s \) a state, and \( s' = \text{app}_o(s) \).
Then \( s \models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e)) \) if and only if \( s' \models a \).

Proof.

\( (\Rightarrow) \): Assume \( s \models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e)) \).
Do a case analysis on the two disjuncts.

1. Assume that \( s \models EPC_a(e) \). By Lemma A, we have \( a \in [e]_s \) and hence \( s' \models a \).
2. Assume that \( s \models a \land \neg EPC_{\neg a}(e) \). By Lemma A, we have \( \neg a \notin [e]_s \). Hence \( a \) remains true in \( s' \).
Regressing state variables: correctness

Proof (ctd.)

(⇐): We showed that if the formula is true in s, then a is true in s'. For the second part, we show that if the formula is false in s, then a is false in s'.

- So assume $s \not\models EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$.
- Then $s \models \neg EPC_a(e) \land (\neg a \lor EPC_{\neg a}(e))$ (de Morgan).
- Case distinction: a is true or a is false in s.
  1. Assume that $s \models a$. Now $s \models EPC_{\neg a}(e)$ because $s \models \neg a \lor EPC_{\neg a}(e)$.
     Hence by Lemma A $\neg a \in [e]_s$ and we get $s' \not\models a$.
  2. Assume that $s \not\models a$. Because $s \models \neg EPC_a(e)$, by Lemma A we get $a \notin [e]_s$ and hence $s' \not\models a$.

Therefore in both cases $s' \not\models a$. 
Regression: general definition

We base the definition of regression on formulae $EPC_i(e)$. 

**Definition (general regression)**

Let $\varphi$ be a propositional formula and $o = \langle \chi, e \rangle$ an operator. The regression of $\varphi$ with respect to $o$ is

$$regr_o(\varphi) = \chi \land \varphi_r \land \kappa$$

where

1. $\varphi_r$ is obtained from $\varphi$ by replacing each $a \in A$ by $EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e))$, and
2. $\kappa = \bigwedge_{a \in A} \neg (EPC_a(e) \land EPC_{\neg a}(e))$.

The formula $\kappa$ expresses that operators are only applicable in states where their change sets are consistent.
Regression examples

- \( \text{regr}_{a,b}(b) \equiv a \land (\top \lor (b \land \neg \bot)) \land \top \equiv a \)
- \( \text{regr}_{a,b}(b \land c \land d) \)
  \[ \equiv a \land (\top \lor (b \land \neg \bot)) \land (\bot \lor (c \land \neg \bot)) \land (\bot \lor (d \land \neg \bot)) \land \top \]
  \[ \equiv a \land c \land d \]
- \( \text{regr}_{a,c \succ b}(b) \equiv a \land (c \lor (b \land \neg \bot)) \land \top \equiv a \land (c \lor b) \)
- \( \text{regr}_{a,(c \succ b) \land (b \succ \neg b)}(b) \equiv a \land (c \lor (b \land \neg b)) \land \neg (c \land b) \)
  \[ \equiv a \land c \land \neg b \]
- \( \text{regr}_{a,(c \succ b) \land (d \succ \neg b)}(b) \equiv a \land (c \lor (b \land \neg d)) \land \neg (c \land d) \)
  \[ \equiv a \land (c \lor b) \land (c \lor \neg d) \land (\neg c \lor \neg d) \]
  \[ \equiv a \land (c \lor b) \land \neg d \]
Regression example: binary counter

\[
(\neg b_0 \triangleright b_0) \land \\
((\neg b_1 \land b_0) \triangleright (b_1 \land \neg b_0)) \land \\
((\neg b_2 \land b_1 \land b_0) \triangleright (b_2 \land \neg b_1 \land \neg b_0))
\]

\[
EPC_{b_2}(e) = \neg b_2 \land b_1 \land b_0 \\
EPC_{b_1}(e) = \neg b_1 \land b_0 \\
EPC_{b_0}(e) = \neg b_0 \\
EPC_{\neg b_2}(e) = \bot \\
EPC_{\neg b_1}(e) = \neg b_2 \land b_1 \land b_0 \\
EPC_{\neg b_0}(e) = (\neg b_1 \land b_0) \lor (\neg b_2 \land b_1 \land b_0) \equiv (\neg b_1 \lor \neg b_2) \land b_0
\]

Regression replaces state variables as follows:

\[
\begin{align*}
b_2 & \quad \text{by} \quad (\neg b_2 \land b_1 \land b_0) \lor (b_2 \land \bot) \equiv (b_1 \land b_0) \lor b_2 \\
b_1 & \quad \text{by} \quad (\neg b_1 \land b_0) \lor (b_1 \land \neg (\neg b_2 \land b_1 \land b_0)) \\
& \quad \equiv (\neg b_1 \land b_0) \lor (b_1 \land (b_2 \lor \neg b_0)) \\
b_0 & \quad \text{by} \quad \neg b_0 \lor (b_0 \land \neg ((\neg b_1 \lor \neg b_2) \land b_0)) \equiv \neg b_0 \lor (b_1 \land b_2)
\end{align*}
\]
General regression: correctness

Theorem (correctness of $\text{regr}_o(\varphi)$)

Let $\varphi$ be a formula, $o$ an operator and $s$ a state. Then $s \models \text{regr}_o(\varphi)$ iff $o$ is applicable in $s$ and $\text{app}_o(s) \models \varphi$.

Proof.

Let $o = \langle \chi, e \rangle$. Recall that $\text{regr}_o(\varphi) = \chi \land \varphi_r \land \kappa$, where $\varphi_r$ and $\kappa$ are as defined previously.

If $o$ is inapplicable in $s$, then $s \not\models \chi \land \kappa$, both sides of the “iff” condition are false, and we are done. Hence, we only further consider states $s$ where $o$ is applicable. Let $s' := \text{app}_o(s)$.

We know that $s \models \chi \land \kappa$ (because $o$ is applicable), so the “iff” condition we need to prove simplifies to:

$$s \models \varphi_r \iff s' \models \varphi.$$
General regression: correctness

Proof (ctd.)

To show: \( s \models \varphi_r \) iff \( s' \models \varphi \).

We show that for all formulae \( \psi \), \( s \models \psi_r \) iff \( s' \models \psi \), where \( \psi_r \) is \( \psi \) with every \( a \in A \) replaced by \( EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e)) \).

The proof is by structural induction on \( \psi \).

- **Induction hypothesis** \( s \models \psi_r \) if and only if \( s' \models \psi \).

- **Base cases 1 & 2** \( \psi = \top \) or \( \psi = \bot \): trivial, as \( \psi_r = \psi \).

- **Base case 3** \( \psi = a \) for some \( a \in A \):
  - Then \( \psi_r = EPC_a(e) \lor (a \land \neg EPC_{\neg a}(e)) \).
  - By Lemma B, \( s \models \psi_r \) iff \( s' \models \psi \).
General regression: correctness

Proof (ctd.)

Inductive case 1 \( \psi = \neg \psi' \):

\[
s \models \psi_r \text{ iff } s \models (\neg \psi')_r \text{ iff } s \models \neg (\psi'_r) \text{ iff } s \not\models \psi'_r
\]

iff (IH) \( s' \not\models \psi' \text{ iff } s' \models \neg \psi' \text{ iff } s' \models \psi
\]

Inductive case 2 \( \psi = \psi' \lor \psi'' \):

\[
s \models \psi_r \text{ iff } s \models (\psi' \lor \psi'')_r \text{ iff } s \models \psi'_r \lor \psi''_r
\]

iff \( s \models \psi'_r \text{ or } s \models \psi''_r
\]

iff (IH, twice) \( s' \models \psi' \text{ or } s' \models \psi''
\]

iff \( s' \models \psi' \lor \psi'' \text{ iff } s' \models \psi
\]

Inductive case 3 \( \psi = \psi' \land \psi'' \): Very similar to inductive case 2, just with \( \land \) instead of \( \lor \) and “and” instead of “or”.

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Emptiness and subsumption testing

The following two tests are useful when performing regression searches to avoid exploring unpromising branches:

- Test that $\text{regr}_o(\varphi)$ does not represent the empty set (which would mean that search is in a dead end). For example, $\text{regr}_{\langle a, \neg p \rangle}(p) \equiv a \land \bot \equiv \bot$.

- Test that $\text{regr}_o(\varphi)$ does not represent a subset of $\varphi$ (which would make the problem harder than before). For example, $\text{regr}_{\langle b, c \rangle}(a) \equiv a \land b$.

Both of these problems are NP-hard.
The formula $\text{regr}_{o_1}(\text{regr}_{o_2}(\ldots \text{regr}_{o_{n-1}}(\text{regr}_{o_n}(\varphi))))$ may have size $O(|\varphi||o_1||o_2|\ldots|o_{n-1}||o_n|)$, i.e., the product of the sizes of $\varphi$ and the operators.

⇝ worst-case exponential size $O(m^n)$

**Logical simplifications**

- $\bot \land \varphi \equiv \bot$, $\top \land \varphi \equiv \varphi$, $\bot \lor \varphi \equiv \varphi$, $\top \lor \varphi \equiv \top$
- $a \lor \varphi \equiv a \lor \varphi[\bot/a]$, $\neg a \lor \varphi \equiv \neg a \lor \varphi[\top/a]$, $a \land \varphi \equiv a \land \varphi[\top/a]$, $\neg a \land \varphi \equiv \neg a \land \varphi[\bot/a]$
- idempotency, absorption, commutativity, associativity, …
Restricting formula growth in search trees

Problem  very big formulae obtained by regression

Cause  disjunctivity in the (NNF) formulae
(formulae without disjunctions easily convertible to
small formulae \( l_1 \land \cdots \land l_n \) where \( l_i \) are literals and \( n \)
is at most the number of state variables.)

Idea  handle disjunctivity when generating search trees
Unrestricted regression: search tree example

Unrestricted regression: do not treat disjunctions specially

Goal $\gamma = a \land b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.
**Full splitting: search tree example**

**Full splitting:** always remove all disjunctivity

Goal $\gamma = a \land b$, initial state $I = \{a \mapsto 0, b \mapsto 0, c \mapsto 0\}$.

$(\neg c \lor a) \land b$ in DNF: $(\neg c \land b) \lor (a \land b)$

$\leadsto$ split into $\neg c \land b$ and $a \land b$

\[
\begin{align*}
\neg a \land a & \quad \langle \neg a, b \rangle \\
(a \land b) & \quad \langle b, \neg c \triangleright a \rangle \\
\text{(duplicate of $\gamma$)} & \\
\neg c \land b & \quad \langle b, \neg c \triangleright a \rangle \\
\neg c \land b & \quad \langle b, \neg c \triangleright a \rangle \\
\neg c \land \neg a & \quad \langle \neg a, b \rangle \\
\neg c \land b & \\
\end{align*}
\]

$\gamma = a \land b$
General splitting strategies

Alternatives:

1. Do nothing (unrestricted regression).
2. Always eliminate all disjunctivity (full splitting).
3. Reduce disjunctivity if formula becomes too big.

Discussion:

- With unrestricted regression the formulae may have size that is exponential in the number of state variables.
- With full splitting search tree can be exponentially bigger than without splitting.
- The third option lies between these two extremes.
(Classical) search is a very important planning approach. Search-based planning algorithms differ along many dimensions, including

- search direction (forward, backward)
- what each search node represents
  (a state, a set of states, an operator sequence)

Progression search proceeds forwards from the initial state.

- If we use duplicate detection, each search node corresponds to a unique state.
- If we do not use duplicate detection, each search node corresponds to a unique operator sequence.
Regression search proceeds backwards from the goal.
- Each search node corresponds to a set of states represented by a formula.
- Regression is simple for STRIPS operators.
- The theory for general regression is more complex.
- When applying regression in practice, additional considerations such as when and how to perform splitting come into play.