1 Introduction

Motivation
- Main problem with semantic networks and frames
  ... the lack of formal semantics!
- Disadvantage of simple inheritance networks
  ... concepts are atomic and do not have any structure
  ~ Brachman’s structural inheritance networks (1977)

Structural inheritance networks
- Concepts are defined/described using a small set of well-defined operators
- Distinction between conceptual and object-related knowledge
- Computation of subconcept relation and of instance relation
- Strict inheritance (of the entire structure of a concept): inherited properties cannot be overridden
Systems and applications

- **Systems:**
  - KL-ONE: First implementation of the ideas (1978)
  - then: NIKL, KL-TWO, KRYPTON, KANDOR, CLASSIC, BACK, KRIS, YAK, CRACK...
  - later: FaGT, DLP, RACER 1998
  - currently: FaCT++, RACER, Pellet.

- **Applications:**
  - First, natural language understanding systems,
  - then configuration systems,
  - and information systems,
  - currently, it is one tool for the Semantic Web

- **Languages:** DAML+OIL, now OWL (Web Ontology Language)

Description logics

- Previously also known as KL-ONE-alike languages, frame-based languages, terminological logics, concept languages

- **Description Logics (DL) allow us**
  - to describe concepts using complex descriptions,
  - to introduce the terminology of an application and to structure it (TBox),
  - to introduce objects and relate them to the introduced terminology (ABox),
  - and to reason about the terminology and the objects.

Informal example

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>the opposite of female</td>
</tr>
<tr>
<td>A human</td>
<td>a kind of living entity</td>
</tr>
<tr>
<td>A woman</td>
<td>a human and a female</td>
</tr>
<tr>
<td>A man</td>
<td>a human and a male</td>
</tr>
<tr>
<td>A mother</td>
<td>a woman with at least one child that is a human</td>
</tr>
<tr>
<td>A father</td>
<td>a man with at least one child that is a human</td>
</tr>
<tr>
<td>A parent</td>
<td>a mother or a father</td>
</tr>
<tr>
<td>A grandmother</td>
<td>a woman, with at least one child that is a parent</td>
</tr>
<tr>
<td>A mother-wod</td>
<td>a mother with only male children</td>
</tr>
</tbody>
</table>

Possible Questions:

- Is a grandmother a parent?
- Is Diana a parent?
- Is William a man?
Atomic concepts and roles

- **Concept names:**
  - E.g., Grandmother, Male, ... (in the following usually capitalized)
  - We will use symbols such as $A_1, A_2, \ldots$ for concept names
  - **Semantics:** Monadic predicates $A(\cdot)$ or set-theoretically a subset of the universe $A \subseteq D$.

- **Role names:**
  - In our example, e.g., child. Often we will use names such as has-child or something similar (in the following usually lowercase).
  - Role names are disjoint from concept names
  - **Symbolically:** $\sqcap, \sqcup, \ldots$
  - **Semantics:** Binary relations $t(\cdot, \cdot)$ or set-theoretically $t \subseteq D \times D$.

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Boolean operators

- **Syntax:** let $C$ and $D$ be concept descriptions, then the following are also concept descriptions:
  - $C \cap D$ (concept conjunction)
  - $C \cup D$ (concept disjunction)
  - $\neg C$ (concept negation)

- **Examples:**
  - Human $\cap$ Female
  - Father $\sqcap$ Mother
  - $\neg$ Female

- **FOL semantics:** $C(x) \land D(x)$, $C(x) \lor D(x)$, $\neg C(x)$

- **Set semantics:** $C \cap D$, $C \cup D$, $D \setminus C$}

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Concept and role description

- From (atomic) concept and role names, complex concept and role descriptions can be created
- In our example, e.g., "Human and Female."
- **Symbolically:** $C$ for concept descriptions and $r$ for role descriptions

Which particular constructs are available depends on the chosen description logic!

- **FOL semantics:** A concept description $C$ corresponds to a formula $C(x)$ with the free variable $x$.
  - Similarly with role descriptions $r$: they correspond to formulae $r(x, y)$ with free variables $x, y$.
- **Set semantics:**
  
  $$C^I = \{ d \in D : C(d) \text{ “is true in” } I \}$$
  $$r^I = \{ (d, e) \in D^2 : r(d, e) \text{ “is true in” } I \}$$

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Role restrictions

- **Motivation:**
  - Often we want to describe something by restricting the possible “fillers” of a role, e.g., Mother-\textit{wod}.
  - Sometimes we want to say that there is at least a filler of a particular type, e.g., Grandmother

- **Idea:** Use quantifiers that range over the role-fillers
  - Mother $\sqcap$ has-child.Man
  - Woman $\sqcap$ has-child.Parent

- **FOL semantics:**
  
  $$(\exists r.C)(x) = \exists y (r(x, y) \land C(y))$$
  $$(\forall r.C)(x) = \forall y (r(x, y) \rightarrow C(y))$$

- **Set semantics:**
  
  $$\{ d \in D : \text{ there exist some } e \text{ s.t. } (d, e) \in r^I \land e \in C^I \}$$
  $$\{ d \in D : \text{ for each } e \text{ with } (d, e) \in r^I, e \in C^I \}$$
Cardinality restriction

Motivation:
- Often we want to describe something by restricting the number of possible “fillers” of a role, e.g., a mother with at least 3 children or at most 2 children.

Idea: We restrict the cardinality of the role filler sets:
- Mother $\sqcap \geq 3$ has-child
- Mother $\sqcap \leq 2$ has-child

FOL semantics:
\[
(\geq n \ r)(x) = \exists y_1 \ldots y_n (r(x, y_1) \wedge \ldots \wedge r(x, y_n) \wedge y_1 \neq y_2 \wedge \ldots \wedge y_{n-1} \neq y_n)
\]
\[
(\leq n \ r)(x) = \neg(\geq n+1 \ r)(x)
\]

Set semantics:
\[
(\geq n \ r)^I = \{d \in D : |\{e \in D : r^I(d, e)\}| \geq n\}
\]
\[
(\leq n \ r)^I = D \setminus (\geq n+1 \ r)^I
\]

Inverse roles

Motivation:
- How can we describe the concept “children of rich parents”?

Idea: Define the “inverse” role for a given role (the converse relation)
- has-child$^{-1}$

Example: $\exists$ has-child$^{-1}$ Rich

FOL semantics:
\[
r^{-1}(x, y) = r(y, x)
\]

Set semantics:
\[
(r^{-1})^I = \{(d, e) \in D^2 : (e, d) \in r^I\}
\]

Role composition

Motivation:
- How can we define the role has-grandchild given the role has-child?

Idea: Compose roles (as one can compose binary relations)

has-child $\circ$ has-child

FOL semantics:
\[
(r \circ s)(x, y) = \exists z (r(x, z) \land s(z, y))
\]

Set semantics:
\[
(r \circ s)^I = \{(d, e) \in D^2 : \exists f \text{ s.t. } (d, f) \in r^I \land (f, e) \in s^I\}
\]

Role value maps

Motivation:
- How do we express the concept “women who know all the friends of their children”?

Idea: Relate role filler sets to each other
- Woman $\sqcap$ (has-child $\circ$ has-friend $\sqsubseteq$ knows)

FOL semantics:
\[
(r \sqsubseteq s)(x) = \forall y (r(x, y) \rightarrow s(x, y))
\]

Set semantics: Let $r^I(d) = \{e : r^I(d, e)\}$.
\[
(r \sqsubseteq s)^I = \{d \in D : r^I(d) \subseteq s^I(d)\}
\]

Note: Role value maps lead to undecidability of satisfiability testing of concept descriptions!
In order to introduce new terms, we use two kinds of terminological axioms:

- \( A \equiv C \)
- \( A \subseteq C \)

where \( A \) is a concept name and \( C \) is a concept description.

A terminology or TBox is a finite set of such axioms with the following additional restrictions:

- no multiple definitions of the same symbol such as \( A \equiv C \), \( A \subseteq D \)
- no cyclic definitions (even not indirectly), such as \( A \equiv \forall r . B \), \( B \equiv \exists s . A \)

TBoxes restrict the set of possible interpretations.

FOL semantics:

- \( A \equiv C \) corresponds to \( \forall x (A(x) \leftrightarrow C(x)) \)
- \( A \subseteq C \) corresponds to \( \forall x (A(x) \rightarrow C(x)) \)

Set semantics:

- \( A \equiv C \) corresponds to \( A^I = C^I \)
- \( A \subseteq C \) corresponds to \( A^I \subseteq C^I \)

Non-empty interpretations which satisfy all terminological axioms are called models of the TBox.

In order to state something about objects in the world, we use two forms of assertions:

- \( a : C \)
- \( (a, b) : r \)

where \( a \) and \( b \) are individual names (e.g., ELIZABETH, PHILIP), \( C \) is a concept description, and \( r \) is a role description.

An ABox is a finite set of assertions.
ABoxes: semantics

- **Individual names** are interpreted as elements of the universe under the **unique-name-assumption**, i.e., different names refer to different objects.
- **Assertions** express that an object is an instance of a concept or that two objects are related by a role.
- **FOL semantics:**
  - \( a : C \) corresponds to \( C(a) \)
  - \( (a,b) : r \) corresponds to \( r(a,b) \)
- **Set semantics:**
  - \( a^T \in D \)
  - \( a : C \) corresponds to \( a^T \in C^T \)
  - \( (a,b) : r \) corresponds to \( (a^T,b^T) \in r^T \)
- **Models** of an ABox and of ABox + TBox can be defined analogously to models of a TBox.

Example TBox

- **Male** = \( \neg \text{Female} \)
- **Human** \( \sqsubseteq \text{Living_entity} \)
- **Woman** = **Human** \( \sqcap \text{Female} \)
- **Man** = **Human** \( \sqcap \text{Male} \)
- **Mother** = **Woman** \( \sqcap \exists \text{has-child} \text{.Human} \)
- **Father** = **Man** \( \sqcap \exists \text{has-child} \text{.Human} \)
- **Parent** = **Father** \( \sqcup \text{Mother} \)
- **Grandmother** = **Woman** \( \sqcap \exists \text{has-child} \text{.Parent} \)
- **Mother-without-daughter** = **Mother** \( \sqcap \forall \text{has-child} \text{.Male} \)
- **Mother-with-many-children** = **Mother** \( \sqcap (\geq 3 \text{has-child}) \)

Example ABox

- **CHARLES**: Man
- **DIANA**: Woman
- **EDWARD**: Man
- **ELIZABETH**: Woman
- **ANDREW**: Man
- **DIANA**: Mother-without-daughter
- \( (\text{ELIZABETH}, \text{CHARLES}) \): has-child
- \( (\text{ELIZABETH}, \text{EDWARD}) \): has-child
- \( (\text{ELIZABETH}, \text{ANDREW}) \): has-child
- \( (\text{DIANA}, \text{WILLIAM}) \): has-child
- \( (\text{CHARLES}, \text{WILLIAM}) \): has-child

4 Reasoning Services
Some reasoning services

- Does a description $C$ make sense at all, i.e., is it satisfiable? A concept description $C$ is satisfiable, if there exists an interpretation $I$ such that $C^I \neq \emptyset$.
- Is one concept a specialization of another one, is it subsumed? $C$ is subsumed by $D$ (in symbols $C \sqsubseteq D$) if we have for all interpretations $C^I \subseteq D^I$.
- Is $a$ an instance of a concept $C$? $a$ is an instance of $C$ if for all interpretations, we have $a^I \in C^I$.

*Note:* These questions can be posed with or without a TBox that restricts the possible interpretations.

5 Outlook

- Can we reduce the reasoning services to perhaps just one problem?
- What could be reasoning algorithms?
- What can we say about complexity and decidability?
- What has all that to do with modal logics?
- How can one build efficient systems?

Literature I

Summary: Concept descriptions

<table>
<thead>
<tr>
<th>Abstract</th>
<th>Concrete</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$A^I$</td>
<td></td>
</tr>
<tr>
<td>$C \cap D$ (and $C \cap D$)</td>
<td>$C^I \cap D^I$</td>
<td></td>
</tr>
<tr>
<td>$C \cup D$ (or $C \cup D$)</td>
<td>$C^I \cup D^I$</td>
<td></td>
</tr>
<tr>
<td>$\neg C$ (not $C$)</td>
<td>$D - C^I$</td>
<td></td>
</tr>
<tr>
<td>$\forall r.C$ (all $r. C$)</td>
<td>${d \in D : r_I^I(d) \subseteq C^I}$</td>
<td></td>
</tr>
<tr>
<td>$\exists r$ (some $r$)</td>
<td>${d \in D : r_I^I(d) \not\subseteq \emptyset}$</td>
<td></td>
</tr>
<tr>
<td>$\geq n r$ (atleast $n r$)</td>
<td>${d \in D : r_I^I(d) \geq n}$</td>
<td></td>
</tr>
<tr>
<td>$\leq n r$ (atmost $n r$)</td>
<td>${d \in D : r_I^I(d) \leq n}$</td>
<td></td>
</tr>
<tr>
<td>$\exists r.C$ (some $r. C$)</td>
<td>${d \in D : r_I^I(d) \cap C^I \not\subseteq \emptyset}$</td>
<td></td>
</tr>
<tr>
<td>$\geq n r.C$ (atleast $n r. C$)</td>
<td>${d \in D : r_I^I(d) \cap C^I \geq n}$</td>
<td></td>
</tr>
<tr>
<td>$\leq n r.C$ (atmost $n r. C$)</td>
<td>${d \in D : r_I^I(d) \cap C^I \leq n}$</td>
<td></td>
</tr>
<tr>
<td>$r = s$ (eq $r s$)</td>
<td>${d \in D : r_I^I(d) = s^I(d)}$</td>
<td></td>
</tr>
<tr>
<td>$r \neq s$ (neq $r s$)</td>
<td>${d \in D : r_I^I(d) \neq s^I(d)}$</td>
<td></td>
</tr>
<tr>
<td>$r \subseteq s$ (subset $r s$)</td>
<td>${d \in D : r_I^I(d) \subseteq s^I(d)}$</td>
<td></td>
</tr>
<tr>
<td>$g = h$ (eq $g h$)</td>
<td>${d \in D : g^I(d) = h^I(d) \not\subseteq \emptyset}$</td>
<td></td>
</tr>
<tr>
<td>$g \neq h$ (neq $g h$)</td>
<td>${d \in D : g^I(d) \neq h^I(d) \not\subseteq \emptyset}$</td>
<td></td>
</tr>
<tr>
<td>${i_1, i_2, \ldots i_n}$</td>
<td>(oneof $i_1, \ldots i_n$)</td>
<td>${i_1^I, i_2^I, \ldots i_n^I}$</td>
</tr>
</tbody>
</table>