Exercise 12.1 (Allen’s tractable subclasses, 7 marks)
Allen’s interval calculus has 8192 possible relations. Which of them are contained in the so-called Continuous Endpoint Class (CEP), which of them are in the so-called Endpoint Class (EP)?

Implement a program that checks for each of the 8192 Allen relations whether it belongs to $C$ or to $P$. The program should understand two flags: --continuous to compute a list of all relations in the CEP subclass; --endpoint to compute all relations in the EP subclass.

The output of the program should be given on the standard output and respect the following format. Each relation is given on a new line. Each line consists of a left parenthesis, followed by a whitespace, the list of base relations separated by a whitespace, and closed by a whitespace and a right parenthesis. As shortcuts for the base relations use the following mapping: before: $<$, after: $>$, meets: $m$, meets$^{-1}$: $mi$, overlaps: $o$, overlaps$^{-1}$: $oi$, starts: $s$, starts$^{-1}$: $si$, finishes: $f$, finishes$^{-1}$: $fi$, during: $d$, during$^{-1}$: $di$, equals: $=$. For example, the relation \{during, overlaps, starts\} should be written as:

\[( d o s )\]

Be careful to respect this format. The script checking your answer will consider it wrong if it is not respected.

Source code must be submitted to: hue@informatik.uni-freiburg.de.

Exercise 12.2 (RCC8 Reduction, 3 marks)
Propose a polynomial reduction from the betweenness problem to the RCC8 network consistency problem, that is, an input instance of the betweenness problem should have a solution if and only if its translation into an RCC8 constraint network can be shown to be satisfiable (using path consistency and backtracking search).

Instances of the betweenness problem are formally defined as follows. Let $X = \{a_1, \ldots, a_m\}$ be a set of elements and $\Phi = \{(a^1_i, a^1_j, a^1_k), \ldots, (a^n_i, a^n_j, a^n_k)\}$ be a set of ordered triples. Each triple is a constraint that states that the second element should be between the first and third one. That is, if the elements of $X$ are ordered by some relation $<$, the triple $(a^1_i, a^1_j, a^1_k)$ is satisfied if and only if it holds $a^1_i < a^1_j < a^1_k$ or $a^1_k < a^1_j < a^1_i$ (the triple is violated in the following cases: $a^1_j < a^1_i < a^1_k$, $a^1_j < a^1_k < a^1_i$, $a^1_i < a^1_k < a^1_j$ or $a^1_k < a^1_i < a^1_j$), etc.

A solution of the betweenness problem $(X, \Phi)$ is a total order on the elements of $X$ such that each constraint in $\Phi$ is satisfied.

The betweenness problem, then, is the problem of checking whether a a given instance of the problem has a solution. (The problem is known to be NP-complete).