Exercise 5.1 (Modal logic: Frame properties, 2+1+2)
State for each of the following frame properties an axiom schema that defines it. Prove that your schema is valid in each frame that has the property. Furthermore, state an appropriate frame that does not have the property and in which the axiom is not valid.

(a) \( vRv \) and \( vRw \) implies \( uRw \).
(b) For each \( v \) there exists at most one \( w \) with \( vRw \).
(c) \( uRv \) and \( uRw \) implies that there exists an \( x \) with \( vRx \) and \( wRx \).

Exercise 5.2 (Modal logic: Tableaux rules, 2 + 1 + 2)
Use tableaux to prove or disproof the following statements. Construct a counterexample from the tableau when possible.

(a) \( \Box \Diamond p \leftrightarrow \Box \Box \Diamond p \) is S4-valid.
(b) \( \Diamond \Box p \rightarrow \Box \Diamond p \) is KT5-valid.
(c) \( \Box (\Box p \rightarrow q) \lor \Box (\Box q \rightarrow p) \) is S4-valid.