Exercise 1.1 (Propositional Logic, 1+1+1+3)

(a) Provide a set $F$ containing 3 propositional formulas such that any strict subset of $F$ is satisfiable but $F$ itself is not satisfiable.

(b) Show, by checking all possible interpretations, that the formula $((\neg A \land B \land C) \lor (A \land B) \lor (\neg A \land \neg C) \lor \neg B)$ is valid.

(c) Show, by using resolution, that $A \land B \land C$ is a consequence from $F = \{ \{\neg A, B\}, \{\neg B, C\}, \{A, \neg C\}, \{A, B, C\} \}$.

(d) Prove that there is no polynomial algorithm that transforms an arbitrary propositional logic formula into a logically equivalent formula in CNF.

_Hint:_ Find a family of formulae in DNF with $n$ variables such that every equivalent formula in CNF must consist of an exponential number of clauses.

Exercise 1.2 (Knowledge representation, 2+2)

Something bad happened at the X-mansion. Halloween went bad and some drunk X-men locked Prof. Xavier into a cell. The guilty people know the code which allows to open the cell but don’t want to give themselves up. Prof. Xavier decided to ask each of the following X-man to give a statement about who was involved. Locked in his special cell his powers are softened and he can only force not guilty people to tell the truth and guilty people to lie.

- Cyclope: "Emma Frost is guilty and Anole is not guilty".
- Emma Frost: "Gambit and Legion are guilty".
- Anole: "Shadowcat is guilty and Legion is not guilty".
- Gambit: "Wolverine is guilty and Prodigy is not guilty".
- Iceberg: "Emma Frost and Storm are not guilty".
- Legion: "Iceberg is guilty and Wolverine is not guilty".
- Malicia: "Prodigy is guilty and Cyclope is not guilty".
- Prodigy: "Emma Frost and Gambit are guilty".
- Shadowcat: "Cyclope is guilty and Iceberg is not guilty".
- Storm: "Gambit is guilty and Malicia is not guilty".
- Vega: "Malicia is guilty and Shadowcat is not guilty".
- Wolverine: "Storm and Vega are guilty".

After gathering all these informations, Prof. Xavier can differentiate the guilty people from the innocent ones. Provide a representation of the problem using propositional logic and give the set of guilty people.