

Principles of AI Planning

6. Planning as search: search algorithms

Albert-Ludwigs-Universität Freiburg



**UNI
FREIBURG**

Bernhard Nebel and Robert Mattmüller

November 13th, 2013



Introduction to search algorithms for planning

Introduction

- Nodes and states
- Search for planning
- Common procedures

Uninformed search

Heuristic search

Summary

Our plan for the next lectures



Choices to make:

- 1 search direction: progression/regression/both
~~> previous chapter
- 2 search space representation: states/sets of states
~~> previous chapter
- 3 search algorithm: uninformed/heuristic; systematic/local
~~> **this chapter**
- 4 search control: heuristics, pruning techniques
~~> next chapters

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary



- Search algorithms are used to find solutions (plans) for **transition systems** in general, not just for planning tasks.
- Planning is **one application** of search among many.
- In this chapter, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.
- Most of this is review of material that should be known (details: Russell and Norvig's textbook).

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

In search, one distinguishes:

- **search states s** \rightsquigarrow states (vertices) of the transition system
- **search nodes σ** \rightsquigarrow search states plus information on where/when/how they are encountered during search

What is in a search node?

Different search algorithms store different information in a search node σ , but typical information includes:

- **$state(\sigma)$** : associated search state
- **$parent(\sigma)$** : pointer to search node from which σ is reached
- **$action(\sigma)$** : action leading from $state(parent(\sigma))$ to $state(\sigma)$
- **$g(\sigma)$** : cost of σ (length of path from the root node)

For the root node, $parent(\sigma)$ and $action(\sigma)$ are undefined.

[Introduction](#)

[Nodes and states](#)

[Search for planning](#)

[Common
procedures](#)

[Uninformed
search](#)

[Heuristic
search](#)

[Summary](#)

Search states \neq (planning) states:

- **Search states** don't have to correspond to **states** in the planning sense.
 - progression: search states \approx **(planning) states**
 - regression: search states \approx **sets of states** (formulae)
- Search algorithms for planning where search states are planning states are called **state-space search** algorithms.
- Strictly speaking, regression is **not** an example of state-space search, although the term is often used loosely.
- However, we will put the emphasis on progression, which is almost always state-space search.

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

A general search algorithm can be applied to any transition system for which we can define the following three operations:

- `init()`: generate the **initial state**
- `is-goal(s)`: test if a given state is a **goal state**
- `succ(s)`: generate the set of **successor states** of state s , along with the **operators** through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a **search space** (a very similar notion to a transition system).

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for progression search

states: all states of Π (assignments to A)

- $\text{init}() = I$
- $\text{is-goal}(s) = \begin{cases} \text{true} & \text{if } s \models \gamma \\ \text{false} & \text{otherwise} \end{cases}$
- $\text{succ}(s) = \{ \langle o, s' \rangle \mid \text{applicable } o \in O, s' = \text{app}_o(s) \}$

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for regression search

states: all formulae over A (how many?)

- $\text{init}() = \gamma$
- $\text{is-goal}(\varphi) = \begin{cases} \text{true} & \text{if } I \models \varphi \\ \text{false} & \text{otherwise} \end{cases}$
- $\text{succ}(\varphi) = \{ \langle o, \varphi' \rangle \mid o \in O, \varphi' = \text{regr}_o(\varphi), \varphi' \text{ is satisfiable} \}$
(modified if splitting is used)

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

uninformed search vs. heuristic search:

- **uninformed search algorithms** only use the basic ingredients for general search algorithms
- **heuristic search algorithms** additionally use **heuristic functions** which estimate how close a node is to the goal

systematic search vs. local search:

- **systematic algorithms** consider a large number of search nodes simultaneously
- **local search algorithms** work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are **crossbreeds** (e. g., enforced hill-climbing)

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

Classification: what works where in planning?



uninformed vs. heuristic search:

- For **satisficing** planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For **optimal** planning, the difference is less pronounced.

systematic search vs. local search:

- For **satisficing** planning, the most successful algorithms are somewhere between the two extremes.
- For **optimal** planning, systematic algorithms are required.



Before we describe the different search algorithms, we introduce three procedures used by all of them:

- **make-root-node:** Create a search node without parent.
- **make-node:** Create a search node for a state generated as the successor of another state.
- **extract-solution:** Extract a solution from a search node representing a goal state.

Introduction

Nodes and states

Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

make-root-node: Create a search node without parent.

Procedure make-root-node

```
def make-root-node(s):  
     $\sigma$  := new node  
    state( $\sigma$ ) := s  
    parent( $\sigma$ ) := undefined  
    action( $\sigma$ ) := undefined  
    g( $\sigma$ ) := 0  
    return  $\sigma$ 
```

Introduction

Nodes and states
Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

make-node: Create a search node for a state generated as the successor of another state.

Procedure make-node

```
def make-node( $\sigma$ ,  $o$ ,  $s$ ):  
     $\sigma' := \mathbf{new}$  node  
    state( $\sigma'$ ) :=  $s$   
    parent( $\sigma'$ ) :=  $\sigma$   
    action( $\sigma'$ ) :=  $o$   
     $g(\sigma') := g(\sigma) + 1$   
    return  $\sigma'$ 
```

Introduction

Nodes and states
Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary

extract-solution: Extract a solution from a search node representing a goal state.

Procedure extract-solution

```
def extract-solution( $\sigma$ ):  
    solution := new list  
    while parent( $\sigma$ ) is defined:  
        solution.push-front(action( $\sigma$ ))  
         $\sigma$  := parent( $\sigma$ )  
    return solution
```

Introduction

Nodes and states
Search for planning

Common
procedures

Uninformed
search

Heuristic
search

Summary



Uninformed search algorithms

Introduction

Uninformed search

Breadth-first w/o
duplicate detection

Breadth-first with
duplicate detection

Random walk

Heuristic
search

Summary

- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

Popular uninformed systematic search algorithms:

- **breadth-first search**
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:

- **random walk**

[Introduction](#)

[Uninformed search](#)

Breadth-first w/o
duplicate detection

Breadth-first with
duplicate detection

Random walk

[Heuristic search](#)

[Summary](#)

Breadth-first search without duplicate detection



Breadth-first search

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
while not queue.empty():
     $\sigma$  = queue.pop-front()
    if is-goal(state( $\sigma$ )):
        return extract-solution( $\sigma$ )
    for each  $\langle o, s \rangle \in \text{succ}(\text{state}(\sigma))$ :
         $\sigma' := \text{make-node}(\sigma, o, s)$ 
        queue.push-back( $\sigma'$ )
return unsolvable
```

- Possible improvement: duplicate detection (see next slide).
- Another possible improvement: test if σ' is a goal node; if so, terminate immediately. (We don't do this because it obscures the similarity to some of the later algorithms.)

Introduction

Uninformed search

Breadth-first w/o duplicate detection

Breadth-first with duplicate detection

Random walk

Heuristic search

Summary

Breadth-first search with duplicate detection



Breadth-first search with duplicate detection

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed :=  $\emptyset$ 
while not queue.empty():
     $\sigma$  = queue.pop-front()
    if state( $\sigma$ )  $\notin$  closed:
        closed := closed  $\cup$  {state( $\sigma$ )}
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in \text{succ}(\text{state}(\sigma))$ :
             $\sigma' := \text{make-node}(\sigma, o, s)$ 
            queue.push-back( $\sigma'$ )
return unsolvable
```

Introduction

Uninformed
search

Breadth-first w/o
duplicate detection

Breadth-first with
duplicate detection

Random walk

Heuristic
search

Summary

Breadth-first search with duplicate detection



Breadth-first search with duplicate detection

```
queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed :=  $\emptyset$ 
while not queue.empty():
     $\sigma$  = queue.pop-front()
    if state( $\sigma$ )  $\notin$  closed:
        closed := closed  $\cup$  {state( $\sigma$ )}
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in$  succ(state( $\sigma$ )):
             $\sigma' :=$  make-node( $\sigma, o, s$ )
            queue.push-back( $\sigma'$ )
return unsolvable
```

Introduction

Uninformed
search

Breadth-first w/o
duplicate detection

Breadth-first with
duplicate detection

Random walk

Heuristic
search

Summary

Random walk

```
 $\sigma := \text{make-root-node}(\text{init}())$   
forever:  
  if  $\text{is-goal}(\text{state}(\sigma))$ :  
    return  $\text{extract-solution}(\sigma)$   
  Choose a random element  $\langle o, s \rangle$  from  $\text{succ}(\text{state}(\sigma))$ .  
   $\sigma := \text{make-node}(\sigma, o, s)$ 
```

- The algorithm usually does not find any solutions, unless almost every sequence of actions is a plan.
- Often, it runs indefinitely without making progress.
- It can also fail by reaching a **dead end**, a state with no successors. This is a weakness of many local search approaches.

Introduction

Uninformed
search

Breadth-first w/o
duplicate detection

Breadth-first with
duplicate detection

Random walk

Heuristic
search

Summary



Heuristic search algorithms

Introduction

Uninformed
search

**Heuristic
search**

Heuristics

Systematic search

Local search

Summary



- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular systematic heuristic search algorithms:

- greedy best-first search
- A^*
- weighted A^*
- IDA *
- depth-first branch-and-bound search
- ...

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary



- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic local search algorithms:

- hill-climbing
- enforced hill-climbing
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- ...

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Heuristic search: idea



Introduction

Uninformed
search

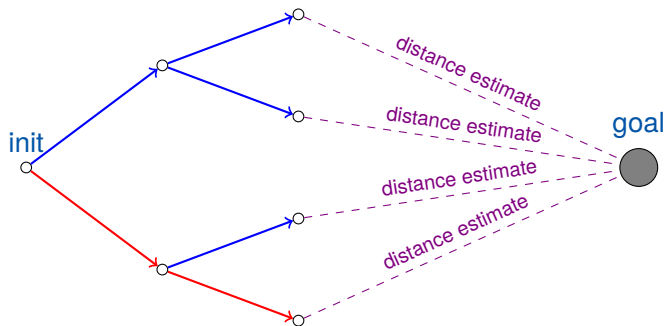
Heuristic
search

Heuristics

Systematic search

Local search

Summary





A **heuristic search algorithm** requires one more operation in addition to the definition of a search space.

Definition (heuristic function)

Let Σ be the set of nodes of a given search space.

A **heuristic function** or **heuristic** (for that search space) is a function $h : \Sigma \rightarrow \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the **heuristic estimate** or **heuristic value** of heuristic h for node σ . It is supposed to estimate the distance from σ to the nearest goal node.

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

What exactly is a heuristic estimate?



What does it mean that h “estimates the goal distance”?

- For most heuristic search algorithms, h does not need to have any strong properties for the algorithm to work (= be correct and complete).
- However, the **efficiency** of the algorithm closely relates to how accurately h reflects the actual goal distance.
- For some algorithms, like A^* , we can prove strong formal relationships between properties of h and properties of the algorithm (optimality, dominance, run-time for bounded error, ...)
- For other search algorithms, “it works well in practice” is often as good an analysis as one gets.

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Heuristics applied to nodes or states?



- Most texts apply heuristic functions to **states**, not **nodes**.
- This is slightly **less general** than our definition:
 - Given a state heuristic h , we can define an equivalent node heuristic as $h'(\sigma) := h(\text{state}(\sigma))$.
 - The opposite is not possible. (Why not?)
- There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on **how** we ended up in a given state s ?
- We call heuristics which don't just depend on $\text{state}(\sigma)$ **pseudo-heuristics**.
- In practice there are sometimes good reasons to have the heuristic value depend on the generating path of σ (e. g., **landmark pseudo-heuristic**, Richter et al. 2008).

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary



Let Σ be the set of nodes of a given search space.

Definition (optimal/perfect heuristic)

The **optimal** or **perfect heuristic** of a search space is the heuristic h^* which maps each search node σ to the length of a shortest path from $state(\sigma)$ to any goal state.

Note: $h^*(\sigma) = \infty$ iff no goal state is reachable from σ .

A heuristic h is called

- **safe** if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- **goal-aware** if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- **admissible** if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- **consistent** if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that σ' is a successor of σ

Relationships?

Greedy best-first search (with duplicate detection)

```
open := new min-heap ordered by ( $\sigma \mapsto h(\sigma)$ )
open.insert(make-root-node(init()))
closed :=  $\emptyset$ 
while not open.empty():
     $\sigma$  = open.pop-min()
    if state( $\sigma$ )  $\notin$  closed:
        closed := closed  $\cup$  {state( $\sigma$ )}
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in$  succ(state( $\sigma$ )):
             $\sigma' :=$  make-node( $\sigma, o, s$ )
            if  $h(\sigma') < \infty$ :
                open.insert( $\sigma'$ )
return unsolvable
```

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Properties of greedy best-first search



- one of the three most commonly used algorithms for satisficing planning
- **complete** for safe heuristics (due to duplicate detection)
- **suboptimal** unless h satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of h (e. g., scaling with a positive constant or adding a constant)

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

A* (with duplicate detection and reopening)

open := **new** min-heap ordered by $(\sigma \mapsto g(\sigma) + h(\sigma))$

open.insert(make-root-node(*init*()))

closed := \emptyset

distance := \emptyset

while not *open.empty*():

$\sigma = \text{open.pop-min}()$

if *state*(σ) \notin *closed* **or** $g(\sigma) < \text{distance}(\text{state}(\sigma))$:

closed := *closed* \cup {*state*(σ)}

distance(*state*(σ)) := $g(\sigma)$

if *is-goal*(*state*(σ)):

return *extract-solution*(σ)

for each $\langle o, s \rangle \in \text{succ}(\text{state}(\sigma))$:

$\sigma' := \text{make-node}(\sigma, o, s)$

if $h(\sigma') < \infty$: *open.insert*(σ')

return unsolvable

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

A* example

Example



Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

0+3



/

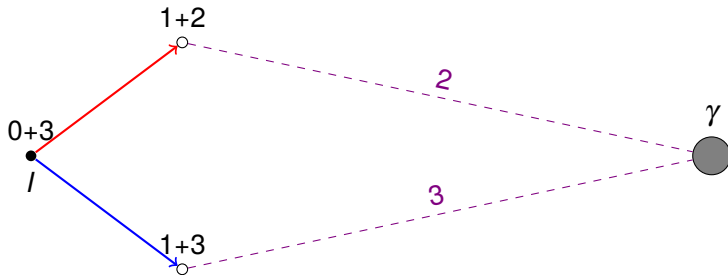
3

γ



A* example

Example



Introduction

Uninformed
search

Heuristic
search

Heuristics

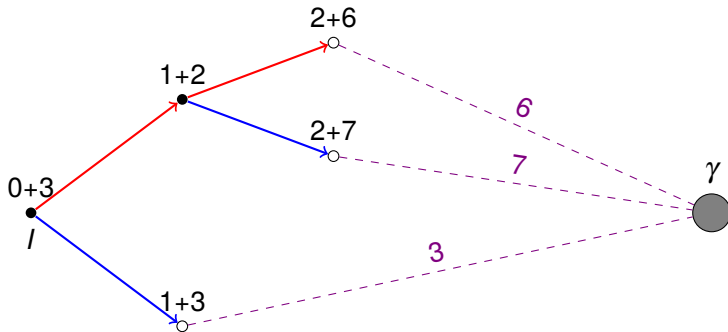
Systematic search

Local search

Summary

A* example

Example



Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

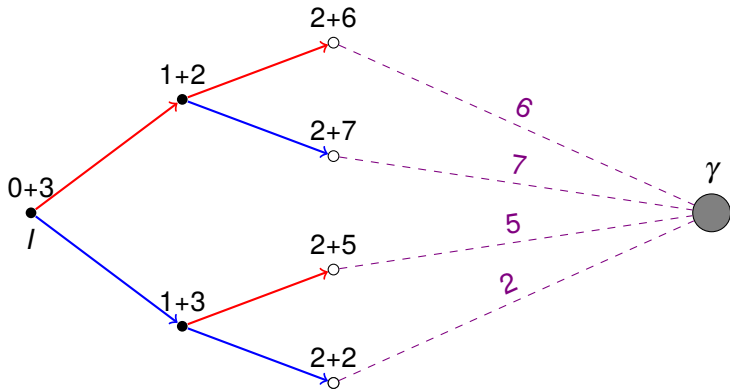
Summary

A* example

Example



UNI
FREIBURG



Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

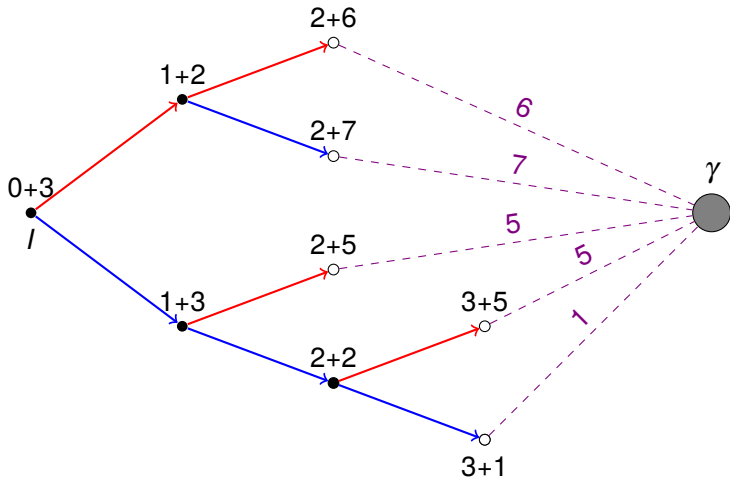
Summary

A* example

Example



UNI
FREIBURG



Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary



- **f value** of a node: defined by $f(\sigma) := g(\sigma) + h(\sigma)$
- **generated nodes**: nodes inserted into *open* at some point
- **expanded nodes**: nodes σ popped from *open* for which the test against *closed* and *distance* succeeds
- **reexpanded nodes**: expanded nodes for which $state(\sigma) \in closed$ upon expansion (also called **reopened nodes**)

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- **complete** for safe heuristics (even without duplicate detection)
- **optimal** if h is admissible (even without duplicate detection)
- never reopens nodes if h is consistent

Implementation notes:

- in the heap-ordering procedure, it is considered a good idea to break ties in favour of lower h values
- can simplify algorithm if we know that we only have to deal with consistent heuristics
- common, hard to spot bug: test membership in *closed* at the wrong time

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Weighted A* (with duplicate detection and reopening)

```
open := new min-heap ordered by ( $\sigma \mapsto g(\sigma) + W \cdot h(\sigma)$ )
open.insert(make-root-node(init()))
closed :=  $\emptyset$ 
distance :=  $\emptyset$ 
while not open.empty():
     $\sigma$  = open.pop-min()
    if state( $\sigma$ )  $\notin$  closed or  $g(\sigma) < \text{distance}(\text{state}(\sigma))$ :
        closed := closed  $\cup$  {state( $\sigma$ )}
        distance( $\sigma$ ) :=  $g(\sigma)$ 
        if is-goal(state( $\sigma$ )):
            return extract-solution( $\sigma$ )
        for each  $\langle o, s \rangle \in \text{succ}(\text{state}(\sigma))$ :
             $\sigma' := \text{make-node}(\sigma, o, s)$ 
            if  $h(\sigma') < \infty$ : open.insert( $\sigma'$ )
return unsolvable
```

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

The **weight** $W \in \mathbb{R}_0^+$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like A^*
- for $W \rightarrow \infty$, behaves like greedy best-first search

Properties:

- one of the most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to A^* , replacing **optimal** with **bounded suboptimal**: generated solutions are at most a factor W as long as optimal ones

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Hill-climbing

```
 $\sigma := \text{make-root-node}(\text{init}())$   
forever:  
  if  $\text{is-goal}(\text{state}(\sigma))$ :  
    return  $\text{extract-solution}(\sigma)$   
   $\Sigma' := \{ \text{make-node}(\sigma, o, s) \mid \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)) \}$   
   $\sigma :=$  an element of  $\Sigma'$  minimizing  $h$  (random tie breaking)
```

- can easily get stuck in **local minima** where immediate improvements of $h(\sigma)$ are not possible
- many variations: tie-breaking strategies, restarts

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Enforced hill-climbing: procedure improve

```
def improve( $\sigma_0$ ):  
    queue := new fifo-queue  
    queue.push-back( $\sigma_0$ )  
    closed :=  $\emptyset$   
    while not queue.empty():  
         $\sigma$  = queue.pop-front()  
        if state( $\sigma$ )  $\notin$  closed:  
            closed := closed  $\cup$  {state( $\sigma$ )}  
            if h( $\sigma$ ) < h( $\sigma_0$ ):  
                return  $\sigma$   
            for each  $\langle o, s \rangle \in$  succ(state( $\sigma$ )):  
                 $\sigma'$  := make-node( $\sigma, o, s$ )  
                queue.push-back( $\sigma'$ )  
  
    fail
```

\rightsquigarrow breadth-first search for more promising node than σ_0

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

Enforced hill-climbing

```
 $\sigma := \text{make-root-node}(\text{init}())$   
while not  $\text{is-goal}(\text{state}(\sigma))$ :  
     $\sigma := \text{improve}(\sigma)$   
return  $\text{extract-solution}(\sigma)$ 
```

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from σ_0)
- complete for **undirected** search spaces (where the successor relation is symmetric) if $h(\sigma) = 0$ for all goal nodes and only for goal nodes

Introduction

Uninformed
search

Heuristic
search

Heuristics

Systematic search

Local search

Summary

- distinguish: **planning states**, **search states**, **search nodes**
 - **planning state**: situation in the world modelled by the task
 - **search state**: subproblem remaining to be solved
 - In **state-space search** (usually progression search), planning states and search states are identical.
 - In regression search, search states usually describe sets of states (“subgoals”).
 - **search node**: search state + info on “how we got there”
- search algorithms mainly differ in **order of node expansion**
 - **uninformed** vs. **informed** (**heuristic**) search
 - **local** vs. **systematic** search

Introduction

Uninformed
search

Heuristic
search

Summary



- **heuristics**: estimators for “distance to goal node”
 - usually: the more accurate, the better performance
 - desiderata: **safe**, **goal-aware**, **admissible**, **consistent**
 - the ideal: **perfect heuristic h^***
- most common algorithms for **satisficing planning**:
 - **greedy best-first search**
 - **weighted A^***
 - **enforced hill-climbing**
- most common algorithm for **optimal planning**:
 - **A^***