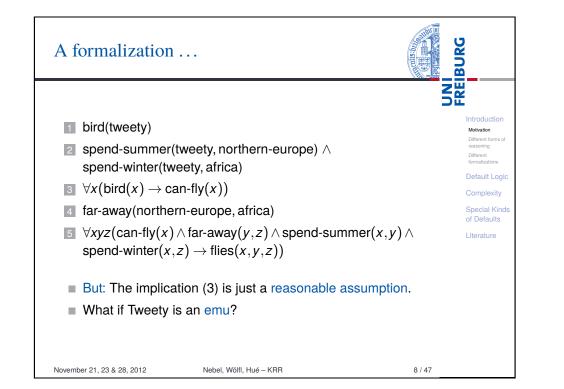
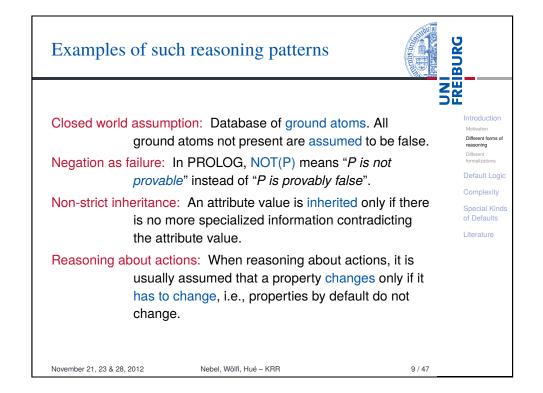


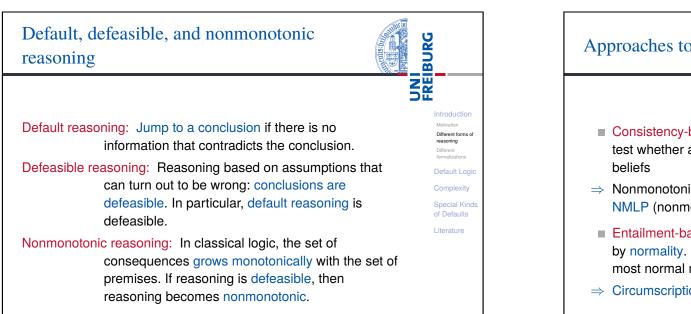
Defaults in knowledge bases	BURG
Often we use default assumptions when definite information is not available or when we want to fix a standard value:	Introduction Motivation Different forms of reasoning
<ol> <li>employee(anne)</li> <li>employee(bert)</li> <li>employee(carla)</li> </ol>	Different formalizations Default Logic Complexity Special Kinds
<ul> <li>employee(detlef)</li> <li>employee(thomas)</li> <li>onUnpaidMPaternityLeave(thomas)</li> </ul>	of Defaults
<ul> <li>Image: Second and the second s</li></ul>	
<b>Typically:</b> employee(X) $\rightarrow \neg$ onUnpaidMPaternityLeave(X)	
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Defaults in commo	n sense reasoning		מ מ מ
winter he stays in A Would you expect T How does Tweety g How would you formalize expected answers?	he stays in Northern Europe, in the frica. weety to be able to fly? et from Northern Europe to Africate this in formal logic so that you g	a? et the	Introduction Motivation Different forms of reasoning Different formalizations Default Logic Complexity Special Kinds of Defaults Literature
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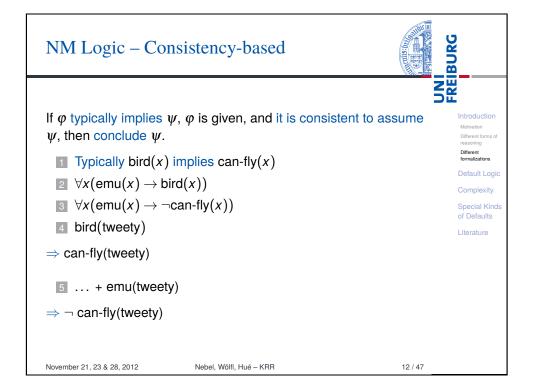




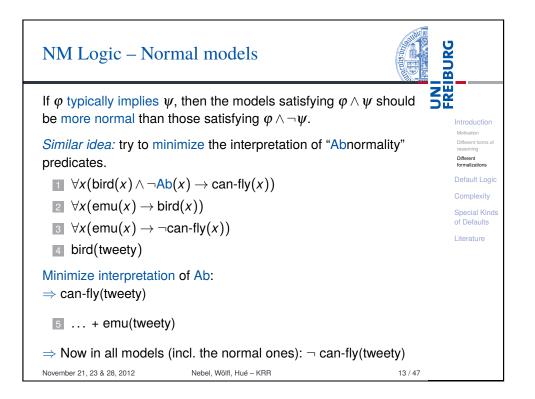
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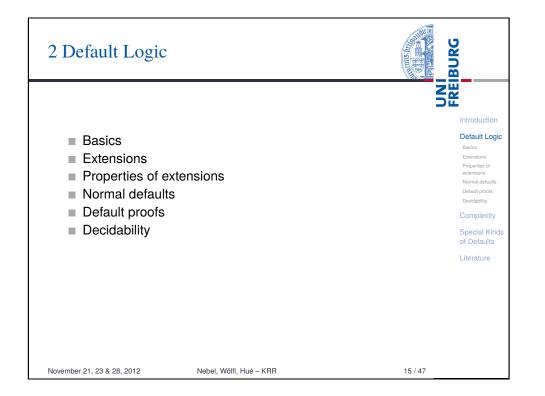
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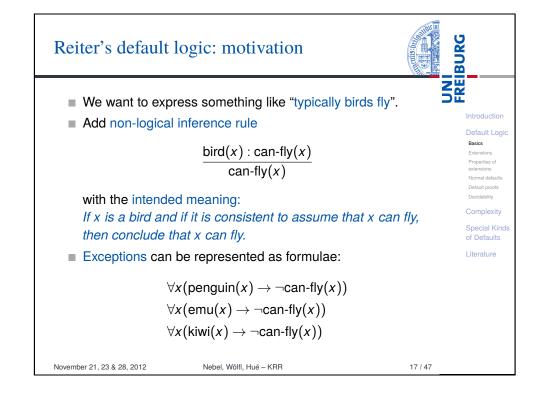
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Approaches to n	onmonotonic reasoni	ing	
		Se	ion
	sed: Extend classical theory assumption is consistent wi	reasoning	IS
	ogics such as DL (default lo otonic logic programming)	Dgic), Complexi Special K of Default	linds
	d on normal models: Mode tailment is determined by co dels only.		9
$\Rightarrow$ Circumscription,	preferential and cumulative	e logics	
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Default Logic – Outline	BURG
1 Introduction	Introduction
2 Default Logic = Basics = Extensions = Properties of extensions = Normal defaults = Default proofs = Decidability	Default Logic Basics Extensions Properties of extensions Normal defaults Default proofs Decidability Complexity Special Kinds of Defaults Literature
3 Complexity of Default Logic	
4 Special Kinds of Defaults November 21, 23 & 28, 2012 Nebel, Wölfl, Hué – KRR	16 / 47

Formal framework	BURG
■ FOL with classical provability relation $\vdash$ and deductive closure: Th( $\Phi$ ) := { $\varphi   \Phi \vdash \varphi$ }	Introduction Default Logic
Default rules: $\frac{\alpha:\beta}{\gamma}$	Basics Extensions Properties of extensions Normal defaults Default proofs
<ul> <li>α: Prerequisite: must have been derived before rule can be applied.</li> <li>β: Consistency condition: the negation may not be derivable.</li> <li>γ: Consequence: will be concluded.</li> </ul>	Decidability Complexity Special Kinds of Defaults
A default rule is closed if it does not contain free variables.	Literature
<ul> <li>(Closed) default theory: A pair (D, W), where D is a countable set of (closed) default rules and W is a countable set of FOL formulae.</li> </ul>	
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## Extensions of default theories



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extensions

Default theories extend the theory given by W using the default rules in D ( $\rightsquigarrow$  extensions). There may be zero, one, or many extensions.

### Example

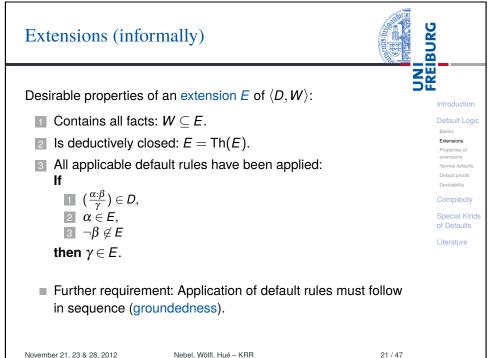
$W = \{a, \neg b\}$	√ ¬ <i>c</i> }
$D = \left\{ \frac{a:b}{b}, \right.$	<u>a: c</u> `
Ъ_{b'	C ,

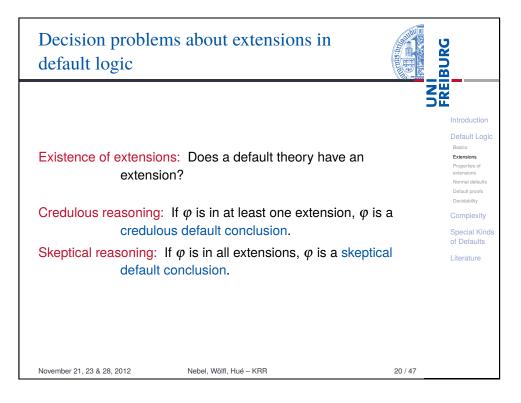
One extension contains b, the other contains c.

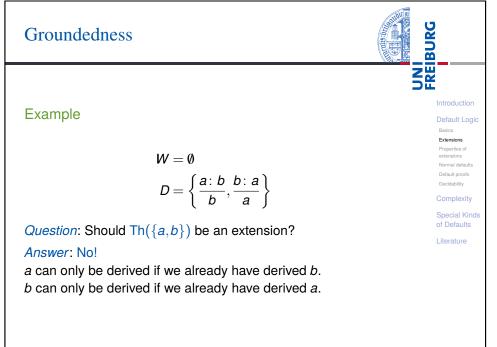
Intuitively, an extension is a set of beliefs resulting from W and D.

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# Extensions (formally)

### Definition

Let  $\Delta = \langle D, W \rangle$  be a closed default theory. Let *E* be any set of closed formulae. Define:

$$E_{0} = W$$
$$E_{i} = \mathsf{Th}(E_{i-1}) \cup \left\{ \gamma \left| \frac{\alpha \colon \beta}{\gamma} \in D, \alpha \in E_{i-1}, \neg \beta \notin E \right. \right\}$$

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Decidability

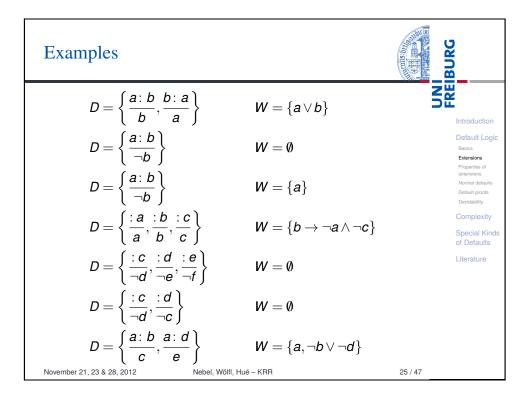
extensions Normal defaults

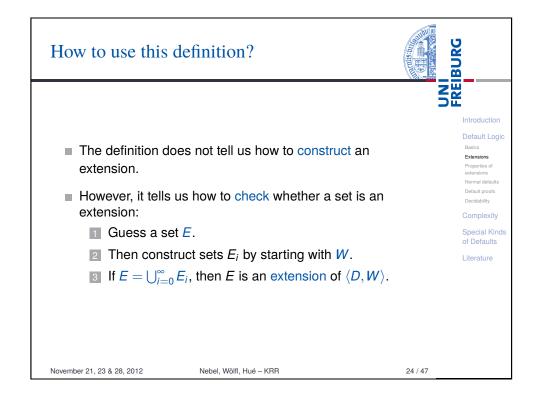
*E* is called an extension of  $\Delta$  if

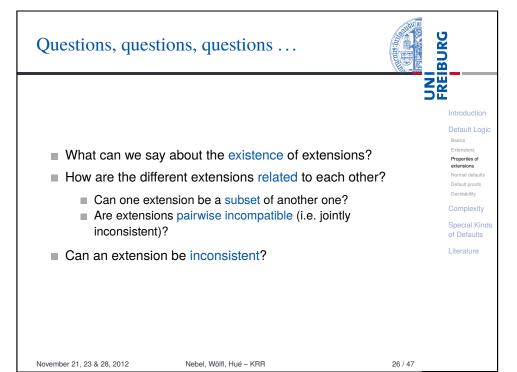
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 $E = \bigcup_{i=0}^{\infty} E_i.$ 

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# Properties of extensions: existence

### Theorem

- If W is inconsistent, there is only one extension.
- A closed default theory (D, W) where all defaults have at least one justification has an inconsistent extension if and only if W is inconsistent.

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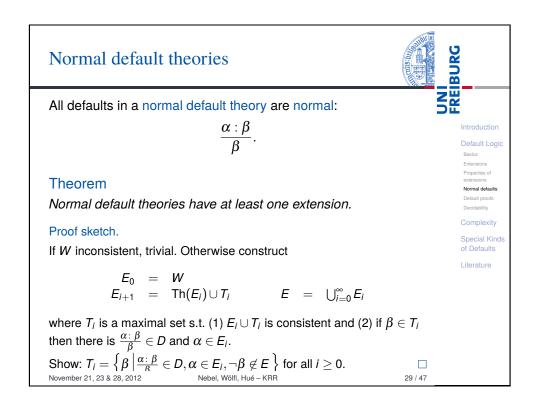
#### Proof idea.

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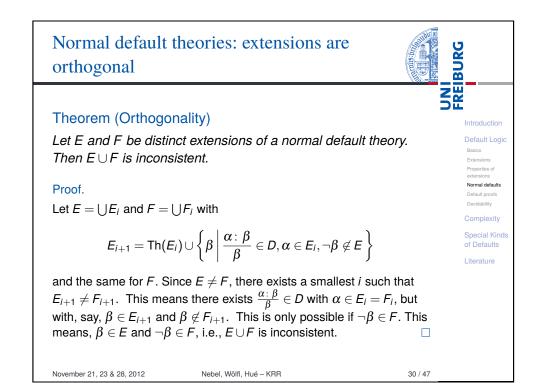
- If W is inconsistent, no default rule is applicable and Th(W) is the only extension.
- 2 Claim 1  $\implies$  the **if**-part.

For **only if**: If *W* is consistent, there is a consistent  $E_i$  s.t.  $E_{i+1}$  is inconsistent. Let  $\{\gamma_1, \ldots, \gamma_n\} = E_{i+1} \setminus \text{Th}(E_i)$  (the conclusions of applied defaults). Now  $\{\neg \beta_1, \ldots, \neg \beta_n\} \cap E = \emptyset$  because otherwise the defaults are not applicable. But this contradicts the inconsistency of *E*.

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Properties of extensions	BURG
Theorem If <i>E</i> and <i>F</i> are extensions of $\langle D, W \rangle$ such that $E \subseteq F$ , then $E = F$	F. Introduction Basics
Proof sketch. $E = \bigcup_{i=0}^{\infty} E_i$ and $F = \bigcup_{i=0}^{\infty} F_i$ . Use induction to show $F_i \subseteq E_i$ . Base case $i = 0$ : Trivially $E_0 = F_0 = W$ . Inductive case $i \ge 1$ : Assume $\gamma \in F_{i+1}$ . Two cases:	Extensions Properties of extensions Normal defaults Default proofs Decidability Complexity Special Kind of Defaults
■ $\gamma \in \text{Th}(F_i)$ implies $\gamma \in \text{Th}(E_i)$ (because $F_i \subseteq E_i$ by IH), and therefore $\gamma \in E_{i+1}$ .	Literature
Cherwise $\frac{\alpha:\beta}{\gamma} \in D$ , $\alpha \in F_i$ , $\neg \beta \notin F$ . However, then we have $\alpha \in E_i$ (because $F_i \subseteq E_i$ ) and $\neg \beta \notin E$ (because of $E \subseteq F$ ), i.e., $\gamma \in E_{i+1}$ .	
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# Default proofs in normal default theories

#### Definition

A default proof of  $\gamma$  in a normal default theory  $\langle D, W \rangle$  is a finite sequence of defaults  $(\delta_i = \frac{\alpha_i : \beta_i}{\beta_i})_{i=1,...,n}$  in *D* such that 1  $W \cup \{\beta_1, \ldots, \beta_n\} \vdash \gamma$ ,

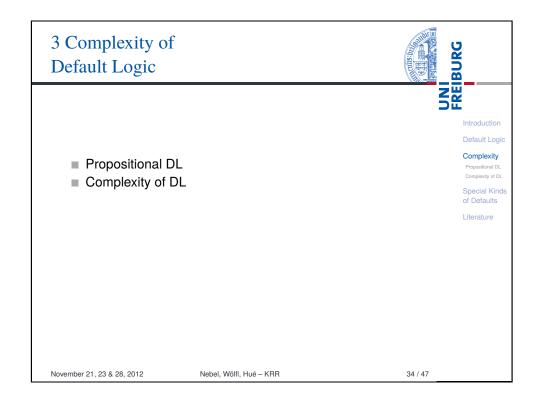
**2**  $W \cup \{\beta_1, \ldots, \beta_n\}$  is consistent, and

**3** 
$$W \cup \{\beta_1, \ldots, \beta_k\} \vdash \alpha_{k+1}$$
, for  $0 \le k \le n-1$ .

### Theorem

Let  $\Delta = \langle D, W \rangle$  be a normal default theory so that W is consistent. Then  $\gamma$  has a default proof in  $\Delta$  if and only if there exists an extension *E* of  $\Delta$  such that  $\gamma \in E$ .

Test 2 (consistency) in the proof procedure suggests that default provability is not even semi-decidable. November 21, 23 & 28, 2012 Nebel, Wölfl, Hué - KRR 31 / 47



# Decidability



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#### Theorem

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Extensions

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Default proofs

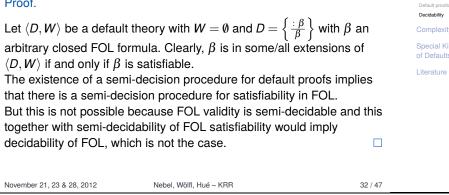
Special Kinds of Defaults

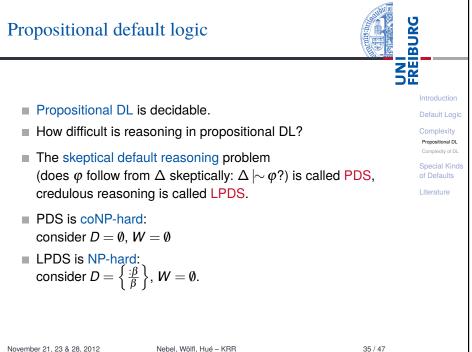
Literature

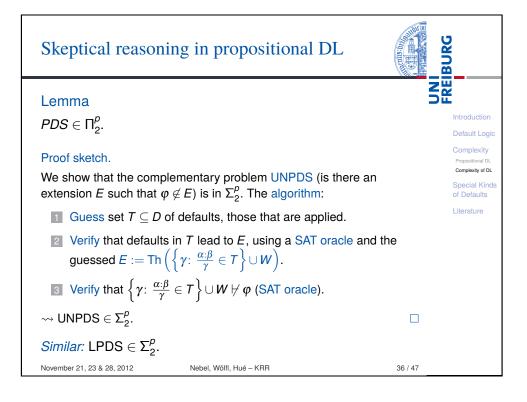
extensions Normal defaults

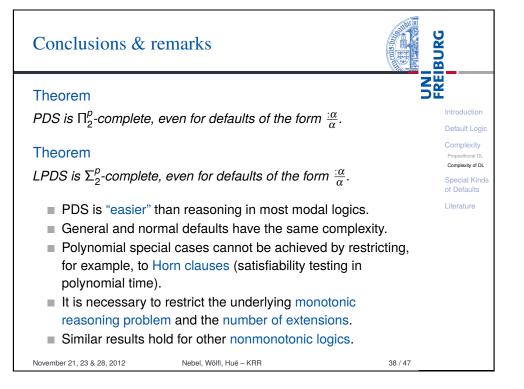
It is not semi-decidable to test whether a formula follows (skeptically or credulously) from a default theory.

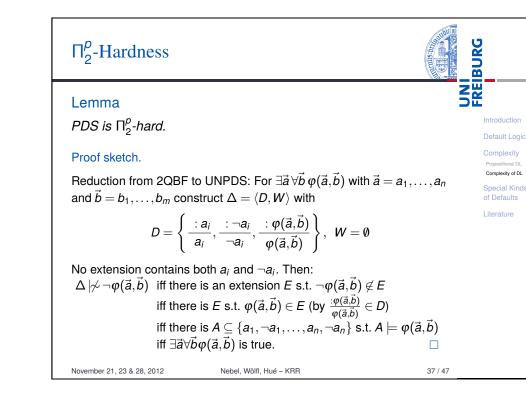
#### Proof.

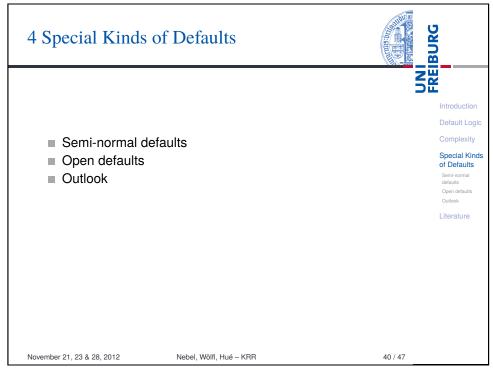


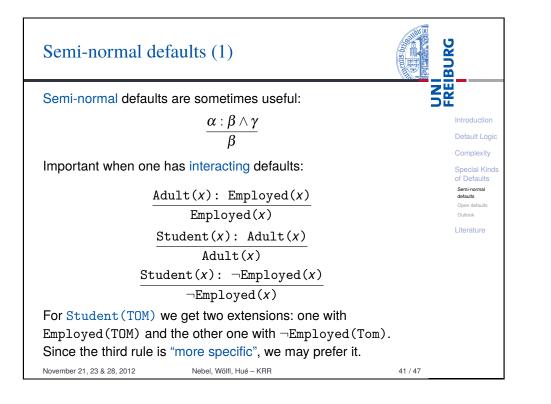


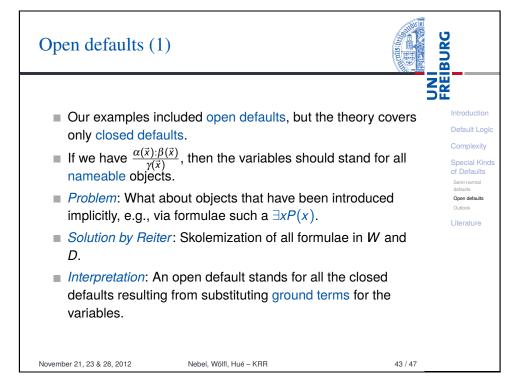


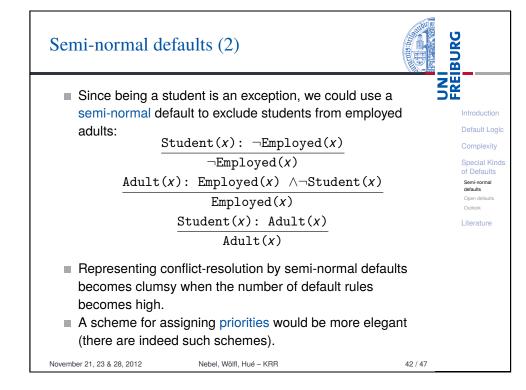


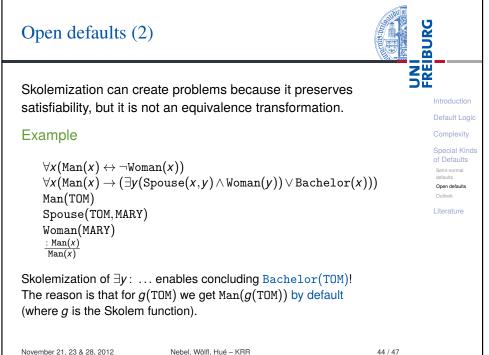












# Open defaults (3)



It is even worse: Logically equivalent theories can have different extensions:

$$W_1 = \{\exists x (P(c,x) \lor Q(c,x))\}$$
$$W_2 = \{\exists x P(c,x) \lor \exists x Q(c,x)\}$$
$$D = \left\{\frac{P(x,y) \lor Q(x,y) \colon R}{R}\right\}$$

Default Logic Complexity Special Kinds of Defaults Semi-normal defaults Open defaults Outlook

Literature

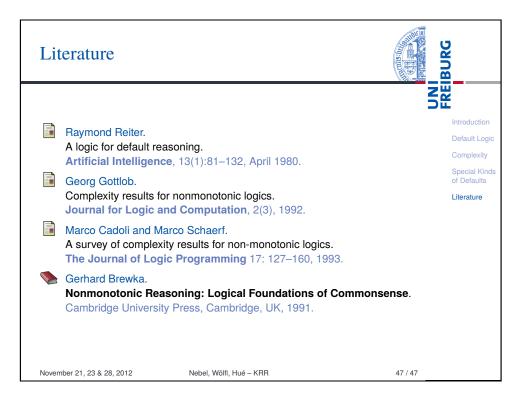
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 $W_1$  and  $W_2$  are logically equivalent. However, the Skolemization of  $W_1$ , symbolically  $s(W_1)$ , is not equivalent with  $s(W_2)$ . The only extension of  $\langle D, W_1 \rangle$  is Th $(s(W_1) \cup R)$ . The only extension of  $\langle D, W_2 \rangle$  is Th $(s(W_2))$ .

*Note*: Skolemization is not the right method to deal with open defaults in the general case.

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Outlook			
•	finition of DL makes sense, c variations and extend the inv		Intro Def
<ul> <li>Extensions can consistency con</li> </ul>	be defined differently (e.g., b aditions).	by remembering	Cor Spe of E
… or by removi	ng the groundedness conditi	ion.	Sen defa Ope
<ul> <li>Open defaults c model-theoretic</li> </ul>	an be handled differently (ma ally).	ore	Out
General proof m	nethods for the finite, decidat	ble case	
Applications of e	default logic:		
<ul><li>Diagnosis</li><li>Reasoning a</li></ul>	about actions		
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