Exercise 12.1 (Formalization in Description Logics, 2+1+1)

(a) Extend the family TBox from the lecture (chapter 11, slide 25) by defining the following concepts:

- Granddaughter
- Brother-in-Law
- Bachelor (Unmarried man)
- Bigamist (Person married to at least two other persons)

Besides the roles and concepts used in the lecture, you may use the atomic role married-to and the (half-)sibling role has-sibling = (has-child$^{-1} \circ$ has-child)$\cap\sim1$.

(b) Prove that in this extended TBox the subsumption relation Bachelor $\sqsubseteq\neg$ Bigamist holds (translate the two concept expressions into their predicate logic semantics and give a proof in predicate logic).

(c) Prove that in the extended TBox the subsumption relation Sister-in-Law $\sqsubseteq$ Granddaughter does not hold by giving a counterexample, i.e., a model of the TBox in which the subsumption does not hold.

Exercise 12.2 (Unfolding, 3)

Specify the unfolding of the concepts Grandmother, Mother-without-daughter, and Bigamist wrt. the TBox in exercise 12.1(a). Determine the primitive components used in your definitions. Provide an initial assignment: use the ABox given in the lecture (chapter 11, slide 26) under the closed world assumption. Finally, specify the full assignments of these three concepts as induced by your initial assignment.

Exercise 12.3 (Realization, 2+2)

Consider the example TBox and ABox (chapter 11, slides 25 & 26) together with the corresponding classification hierarchy (chapter 12, slide 25). The realization of an object $a$ in the ABox consists of the most specialized concepts $C$ of the TBox for which $a : C$ holds.

(a) Give the realization of DIANA, ELIZABETH, CHARLES, EDWARD, ANDREW, and WILLIAM (only results, not proofs).

(b) To algorithmically determine the realization of an object one has to prove a set of statements of the form “$a : C$ follows from the knowledge base” and “$a : C$ does not follow from the knowledge base”. Give a minimal set of such statements that could be used to prove your results from part (a). Note: you may assume that the classification hierarchy has already been proven.