Possibilistic Qualitative Spatial and Temporal Reasoning

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Bachelor/MasterArbeit
Reasoning about qualitative aspects of time and/or space (infinite!) with finitely many qualitative relations between entities of interest e.g., before, during, part of, ... 

Formal relational language $\mathcal{L}$ as a set of relations, usually jointly exhaustive and pairwise disjoint.

Example (8)
Constraint-based Qualitative Reasoning

Reasoning about qualitative aspects of time and/or space (infinite!) with finitely many qualitative relations between entities of interest e.g., *before*, *during*, *part of*, ...

Formal relational language $\mathcal{L}$ as a set of relations, usually jointly exhaustive and pairwise disjoint.

**Example (Allen’s Interval Calculus - AIC)**

<table>
<thead>
<tr>
<th>Relation</th>
<th>Before</th>
<th>Meets</th>
<th>Overlaps</th>
<th>During</th>
<th>Starts</th>
<th>Finishes</th>
<th>Equals</th>
<th>during J</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ before $J$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
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<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
</tr>
<tr>
<td>$I$ meets $J$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
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<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
</tr>
<tr>
<td>$I$ overlaps $J$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
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<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
</tr>
<tr>
<td>$I$ during $J$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
<td>$\overline{I} \underline{J}$</td>
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</tr>
</tbody>
</table>

+ converse relations
Qualitative Constraints

Using relations as constraints
Consider reasoning problems, e.g.:

\[ \varphi := \exists x, y, z \in: x \text{ (PO or DC)} \land y \land y \text{ PO} \land z \land x \text{ DC} \land z \]

\( \leadsto \) satisfiable if there exists a consistent atomic refinement

Consistency?
Inference rules on triples of the form:

\[ \forall x, y, z \in: (x \ R' \ y \land y \ R'' \ z) \rightarrow \neg (x \ R \ z) \]

also known as “composition”

complete set of valid rules is refutation complete for non-dijunctive relations
Motivation

- The previous framework is defined relatively to first-order logic;
- Sometimes preferences can appear between relations: why not a multi-valued logic?

Disconnection > Overlapping
Possibilistic Logic

Possibilistic logic uses two values to characterize an event:

- **Possibility value ($\Pi$):** measuring if the event is compatible with the current knowledge.
- **Necessity value ($N$):** measuring how forced is the event. (Literally how compatible the negation of the event is).

**Merits:**

- Very easy representation of uncertainty: $\Pi(x) = 1$ and $N(x) = 0$.
- Not too intractable ($\mathcal{O}(NP^{\log n})$).
What we are looking for?

- Embed possibilistic logic within QSTR and realize an implementation based on the GQR solver!

Example

$$\varphi := \exists x, y, z \in: \quad \langle x \ PO \ y, 0.4 \rangle \lor \langle x \ DC \ y, 1 \rangle \land \langle y \ PO \ z, 0.9 \rangle \land \langle x \ DC \ z, 1 \rangle$$