Exercise Sheet 8
Due: December 21st, 2012

Exercise 8.1 (Abstraction heuristics, 3 + 2 points)
A state of a 15-puzzle planning task is given as a permutation \langle b, t_1, \ldots, t_{15} \rangle of \{1, \ldots, 16\}, where \( b \) denotes the empty tile (blank) and all other components denote the positions of the tiles.

Let \( T^1 = \{t^1_1, \ldots, t^1_n\}, T^2 = \{t^2_1, \ldots, t^2_m\} \) with \( 1 \leq n, m \leq 14 \) be a partitioning of \( \{t_1, \ldots, t_{15}\} \) (i.e., \( T^1 \cup T^2 = \{t_1, \ldots, t_{15}\} \) and \( T^1 \cap T^2 = \emptyset \)). Consider the following abstractions:

- \( \alpha_1(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t^1_1, \ldots, t^1_m \rangle \)
- \( \alpha_2(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t^2_1, \ldots, t^2_n \rangle \)
- \( \alpha_3(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t^1_1, \ldots, t^1_m \rangle \)
- \( \alpha_4(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t^2_1, \ldots, t^2_n \rangle \)

For \( 1 \leq i \leq 4 \), the heuristic estimates of \( h_i \) are equal to lengths of optimal plans in the respective abstractions (e.g., \( h_i(s) = h^*(\alpha_i(s)) \)). Show that:

(a) \( h_1 + h_2 \) is not admissible.
(b) \( h_3 + h_4 \) is admissible.

Hint: A heuristic is admissible if it is goal-aware and consistent.

Exercise 8.2 (Syntactic projections vs. projections, 3 + 2 points)
Let \( \Pi \) be a planning task in finite-domain representation and let \( P \) be a pattern for \( \Pi \). Show that:

(a) If \( \Pi \) is a SAS\(^+\) planning task that is not trivially unsolvable and that has no trivially inapplicable operators, then \( T(\Pi|P) \preceq T(\Pi)^{\pi_P} \).

(b) If there are conditional effects in \( \Pi \), then it is not true in general that \( T(\Pi|P) \preceq T(\Pi)^{\pi_P} \).

Note: The exercise sheets may and should be worked on in groups of two students. Please state both names on your solution (this also holds for submissions by e-mail).