Principles of AI Planning

20. Expressive power

Bernhard Nebel and Robert Mattmüller

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Motivation
Expressive power is the motivation for designing new planning languages.

Often there is the question: **Syntactic sugar or essential feature?**

Compiling away or change planning algorithm?

If a feature can be compiled away, then it is apparently only **syntactic sugar**.

Sometimes, however, a compilation can lead to much larger planning domain descriptions or to much longer plans.

This means the planning algorithm will probably choke, i.e., it cannot be considered as a **compilation**.
Example: DNF Preconditions

Assume we have **DNF preconditions** in STRIPS operators.

This can be compiled away as follows:

Split each operator with a DNF precondition $c_1 \vee \ldots \vee c_n$ into $n$ operators with the same effects and $c_i$ as preconditions.

\[ \iff \text{If there exists a plan for the original planning task there is one for the new planning task and vice versa} \]

\[ \Rightarrow \text{The planning task has almost the same size} \]

\[ \Rightarrow \text{The shortest plans have the same size} \]
Example: Conditional effects

- Can we compile away **conditional effects** to STRIPS?
- Example operator: $\langle a, b \triangleright d \land \neg c \triangleright e \rangle$
- Can be translated into four operators:
  $\langle a \land b \land c, d \rangle, \langle a \land b \land \neg c, d \land e \rangle, \ldots$
- Plan existence and plan size are identical
- **Exponential blowup** of domain description!
  $\rightarrow$ Can this be avoided?
Propositional STRIPS and Variants
Propositional STRIPS and Variants

- In the following we will only consider propositional STRIPS and some variants of it.

- **Planning task:**
  \[ \mathcal{T} = \langle A, I, O, G \rangle. \]

- Often we refer to **domain structures** \( \mathcal{D} = \langle A, O \rangle. \)
Disjunctive Preconditions: Trivial or Essential?

- Kambhampati et al [ECP 97] and Gazen & Knoblock [ECP 97]: Disjunctive preconditions are trivial – since they can be translated to basic STRIPS (DNF-preconditions).
- Bäckström [AIJ 95]: Disjunctive preconditions are probably essential – since they can not easily be translated to basic STRIPS (CNF-preconditions).
- Anderson et al [AIPS 98]: “[D]isjunctive preconditions …are … essential prerequisites for handling conditional effects” $\iff$ conditional effects imply disjunctive preconditions (?) (General Boolean preconditions).
More “Expressive Power”

\[ \text{STRIPS}_N : \text{plain strips with negative literals} \]
\[ \text{STRIPS}_{Bd} : \text{precondition in disjunctive normal form} \]
\[ \text{STRIPS}_{Bc} : \text{precondition in conjunctive normal form} \]
\[ \text{STRIPS}_B : \text{Boolean expressions as preconditions} \]
\[ \text{STRIPS}_C : \text{conditional effects} \]
\[ \text{STRIPS}_{C,N} : \text{conditional effects & negative literals} \]
Ordering Planning Formalisms Partially

Motivation
Propositional STRIPS and Variants
Disjunctive Preconditions: Difficult or Easy?
STRIPS Variants
Partially Ordered STRIPS Variants
Computational Complexity
Expressive Power
Summary
Theorem

PLANEX is PSPACE-complete for STRIPS\textsubscript{N}, STRIPS\textsubscript{C,B}, and for all formalisms “between” the two.

Beweis.

Follows from theorems proved in the previous lecture.
Expressive Power
Measuring Expressive Power

Consider **mappings** between planning problems in different formalisms

- that **preserve**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan “structure”
  - the solutions/plans themselves

- that **are limited**
  - in the size of the result (poly. size)
  - in the computational resources (poly. time)

- that **transform**
  - entire planning instances
  - domain structure and states in isolation
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~> When measuring the expressiveness of planning formalisms, domain structures should be considered independently from states
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When measuring the expressiveness of planning formalisms, domain structures should be considered independently from states.
The Right Method: Compilation Schemes (Simplified)

- Transform **domain structure** $D = \langle A, O \rangle$ (with polynomial blowup) to $D'$ preserving solution existence.
- Only trivial changes to states (independent of operator set).
- Resulting plans $\pi'$ should not grow too much (additive constant, linear growth, polynomial growth).
- Similar to knowledge compilation, with operators as the **fixed part** and initial states & goals as the **varying part**.
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\[ D \xrightarrow{\text{compilation}} D' \]

\[ \pi \xrightarrow{\text{Planning}} \pi' \]
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Similar to knowledge compilation, with operators as the fixed part and initial states & goals as the varying part
\( Y \preceq X \) (\( Y \) is compilable to \( X \))

iff

there exists a compilation scheme from \( Y \) to \( X \).

\( Y \preceq^1 X \): preserving plan size exactly (modulo additive constants)

\( Y \preceq^c X \): preserving plan size linearly (in \( |\pi| \))

\( Y \preceq^p X \): preserving plan size polynomially (in \( |\pi| \) and \(|D|\))

\( Y \preceq^x_p X \): polynomial-time compilability

**Theorem**

For all \( x, y \), the relations \( \preceq^x_y \) are transitive and reflexive.
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$Y \preceq^x_p X$: polynomial-time compilability

**Theorem**

For all $x, y$, the relations $\preceq^x_Y$ are transitive and reflexive.
Shouldn’t we also require that plans in the compiled instance can be **translated back** to the original formalism?

Yes, if we want to use this technique, one should require that!

In all **positive cases**, there was never any problem to translate the plan back.

For the **negative case**, it is easier to prove non-existence.

So, in order to prove negative results, we do not need it, for positive it never had been a problem.

So, similarly to the concentration on **decision problems** when determining complexity, we simplify things here.
A (Trivial) Positive Result: \( \text{STRIPS}_{Bd} \preceq_p^1 \text{STRIPS}_N \)

DNF preconditions can be “compiled away.”

Assume operator \( o = \langle c, e \rangle \) and

\[
c = L_1 \lor \ldots \lor L_k
\]

with \( L_i \) being a conjunction of literals. Create \( k \) operators \( o_i = \langle L_i, e \rangle \)

1. compilation is solution-preserving,
2. \( \mathcal{D}' \) is only polynomially larger than \( \mathcal{D} \),
3. compilation can be computed in polynomial time,
4. resulting plans do not grow at all.

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$\Rightarrow$ $\text{STRIPS}_{Bd} \preceq^1_p \text{STRIPS}_N$
Another Positive Result: $\text{STRIPS}_{C,Bc} \preceq_{\rho}^{C} \text{STRIPS}_{C,N}$

CNF preconditions can be “compiled away” – provided we have already conditional effects.

- Evaluate the truth value of all disjunctions appearing in operators by using a special evaluation operator with conditional effects that make new “clause atoms” true.
- Alternate between executing original operators (clauses replaced by new atoms) and evaluation operators.
- Operator sets grow only polynomially.
- Plans are double as long as the original plans.
- Anderson et al’s conjecture holds in a weak version.

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Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathcal{D}$ with only one (STRIPS$_{C,B}$) operator $o$:

$$\langle \top, (p_1 \triangleright \neg p_1) \land (\neg p_1 \triangleright p_1) \land \ldots \land (p_k \triangleright \neg p_k) \land (\neg p_k \triangleright p_k) \rangle,$$

which “inverts” a given state. For all $(I, G)$ with

$$G = \bigwedge \{ \neg v \mid v \in A, I = v \} \land \bigwedge \{ v \mid v \in A, I \neq v \},$$

there exists a STRIPS$_{C,B}$ one-step plan.

Assume there exists a compilation preserving plan size linearly leading to a STRIPS$_B$ domain structure $\mathcal{D}'$. There are exponentially many possible initial states, but only polynomially many different $c$-step plans for $\mathcal{D}'$. Some STRIPS$_B$ plan $\pi$ is used for different initial states $I_1, I_2$ (for large enough $k$). Let $v$ be a variable with $I_1(v) \neq I_2(v)$.

$\Rightarrow$ In one case, $v$ must be set by $\pi$, in the other case, it must be cleared.

$\Rightarrow$ This is not possible in an unconditional plan.

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which “inverts” a given state. For all $(I, G)$ with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \land \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a STRIPS$_{C,B}$ one-step plan.

Assume there exists a compilation preserving plan size linearly leading to a STRIPS$_B$ domain structure $\mathcal{D}'$. There are exponentially many possible initial states, but only polynomially many different $c$-step plans for $\mathcal{D}'$. Some STRIPS$_B$ plan $\pi$ is used for different initial states $I_1, I_2$ (for large enough $k$). Let $v$ be a variable with $I_1(v) \neq I_2(v)$.

$\Rightarrow$ In one case, $v$ must be set by $\pi$, in the other case, it must be cleared.

$\Rightarrow$ This is not possible in an unconditional plan.

$\Rightarrow$ The transformation is not solution preserving!

$\Rightarrow$ **Conditional effects** cannot be compiled away (if plan size can grow only linearly)
Another Negative Result: $\text{STRIPS}_{Bc} \not\preceq^C \text{STRIPS}_N$

$k$-FISEX: Planning problem with fixed plan length $k$ and varying initial state. Does there exist an initial state leading to a successful $k$-step plan?

1-FISEX is NP-complete for $\text{STRIPS}_{Bc}$ (= SAT).

$k$-FISEX is polynomial for $\text{STRIPS}_N$ (regression analysis)

\[ \leadsto \text{STRIPS}_{Bc} \not\preceq^C \text{STRIPS}_N \text{ (if } P \neq \text{NP)} \]

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

\[ \leadsto \text{Bäckström’s conjecture holds in the compilation framework.} \]
**Another Negative Result:** \( \text{STRIPS}_{Bc} \not\preccurlyeq^{c} \text{STRIPS}_{N} \)

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$\Downarrow \Rightarrow \text{STRIPS}_{Bc} \not<^C_p \text{STRIPS}_N$ (if $P \neq NP$)

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

$\Downarrow \Rightarrow$ Bäckström’s conjecture holds in the compilation framework.
Another Negative Result: $\text{STRIPS}_{Bc} \not\preceq^c \text{STRIPS}_N$

$k$-FISEX: Planning problem with fixed plan length $k$ and varying initial state. Does there exist an initial state leading to a successful $k$-step plan?

1-FISEX is NP-complete for $\text{STRIPS}_{Bc}$ ($= \text{SAT}$).

$k$-FISEX is polynomial for $\text{STRIPS}_N$ (regression analysis).

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $\text{P}/\text{poly}$.

$\leadsto \text{Bäckström’s conjecture holds}$ in the compilation framework.
Another Negative Result: \( \text{STRIPS}_{Bc} \not\leq^c \text{STRIPS}_N \)

\( k\)-FISEX: Planning problem with fixed plan length \( k \) and varying initial state. Does there exist an initial state leading to a successful \( k \)-step plan?

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Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) (= NC\(^1\)).

Conditional effects can simulate only families of circuits with fixed depth (= AC\(^0\)).

The parity function can be expressed in the first framework (NC\(^1\)) while it cannot be expressed in the second (AC\(^0\)).

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⇝ The negative result follows unconditionally!
Boolean Circuits

- We know what **Boolean circuits** are (directed, acyclic graphs with different types of nodes: **and**, **or**, **not**, **input**, **output**)
- **Size of circuit** = number of gates
- **Depth of circuit** = length of longest path from input gate to output gate
- When we want to **recognize formal languages with circuits**, we need a **sequence of circuits** with an increasing number of input gates \( \rightsquigarrow \) **family of circuits**
- Families with polynomial size and poly-log \( (\log^k n) \) depth complexity classes **NC\(^k\)** (Nick’s class)
- **NC** = \( \bigcup_k \text{NC}^k \subseteq P \), the class of problems that can be solved efficiently in parallel
- The class of languages that can be characterized by polynomially sized **Boolean formulae** is identical to **NC\(^1\)**
The classes $\text{AC}^k$

- The classes $\text{NC}^k$ are defined with a fixed fan-in
- If we have unbounded fan-in, we get the classes $\text{AC}^k$
  - gate types: NOT, $n$-ary AND, $n$-ary OR for all $n \geq 2$
- Obviously: $\text{NC}^k \subseteq \text{AC}^k$
- Possible to show: $\text{AC}^{k-1} \subseteq \text{NC}^k$
- The parity language is in $\text{NC}^1$, but not in $\text{AC}^0$!
Accepting languages with families of domain structures with fixed goals

- We will view families of domain structures with fixed goals and fixed size plans as “machines” that accept languages.
- Consider families of poly-sized domain structures in STRIPS$_B$ and use one-step plans for acceptance.
- Obviously, this is the same as using Boolean formulae.
- All languages in NC$^1$ can be accepted in this way.
Represent each operator and then chain the actions together ($O(|O|^c)$ different plans):

\[ p_1^c \quad p_2^c \quad \ldots \quad \ldots \quad p_n^c \]
\[ p_1^0 \quad p_2^0 \quad \ldots \quad \ldots \quad p_n^0 \]
\[ F_1 \quad \ldots \quad F_m \]
Simulating $\text{STRIPS}_{C,N} c$-Step Plans with $\text{AC}^0$ circuits (2)

For each single action (precondition testing (a), conditional effects (b), and the computation of effects (c).
Theorem

\( \text{STRIPS}_B \not\leq^c \text{STRIPS}_{C,N} \).

Beweis.

Assuming \( \text{STRIPS}_B \leq^c \text{STRIPS}_{C,N} \) has the consequence that the underlying compilation scheme could be used to compile a \( \text{NC}^1 \) circuit family into an \( \text{AC}^0 \) circuit family, which is impossible in the general case.
All other potential positive results have been ruled out by our 3 negative results and transitivity.
Summary
Compilation schemes seem to be the right method to measure the relative expressive power of planning formalisms.

Either we get a positive result preserving plan size linearly with a polynomial-time compilation, or we get an impossibility result.

Results are relevant for building planning systems.

CNF preconditions do not add much when we have already conditional effects.

Note: In all cases we can get a positive result if we allow for a polynomial blow-up of the plans.