Exercise 9.1 (Additive patterns and canonical heuristic, 2+2+1 points)
Consider the Sokoban problem given by the picture below. The red circle denotes the agent’s position, the blue squares are boxes, and the green grid cells are the target positions of the boxes (it is irrelevant which box ends up in which target position). The letters only denote the names of the grid cells.

We will model this problem in finite-domain representation using the variables \( \text{position}_p, \text{position}_{s_1}, \text{position}_{s_2}, \text{at-goal}_{s_1}, \text{at-goal}_{s_2}, \text{content}_A, \text{content}_B, \ldots, \text{content}_T \) with the following domains:

- \( D_{\text{position}_p} = D_{\text{position}_{s_1}} = D_{\text{position}_{s_2}} = \{A, B, \ldots, T\} \)
- \( D_{\text{at-goal}_{s_1}} = D_{\text{at-goal}_{s_2}} = \{\text{true, false}\} \)
- \( D_{\text{content}_X} = \{\text{nothing, } p, s_1, s_2\} \)

The initial state is given as

- \( \text{position}_p = S, \text{position}_{s_1} = M, \text{position}_{s_2} = H, \text{at-goal}_{s_1} = \text{at-goal}_{s_2} = \text{false} \)
- \( \text{content}_M = s_2, \text{content}_S = s_1, \text{content}_P = p \)
- \( \text{content}_X = \text{nothing} \) for \( X \in \{A, \ldots, T\} \setminus \{H, M, S\} \)

and the goal formula is \( \text{at-goal}_{s_1} = \text{true} \land \text{at-goal}_{s_2} = \text{true} \). The set of available operators contains the obvious FDR formalizations of all move- and push-actions that are usually available in Sokoban.

Consider the pattern collection \( C \) with the following patterns:

\[
\begin{align*}
P_1 &= \{\text{at-goal}_{s_2}\} \\
P_2 &= \{\text{at-goal}_{s_1}, \text{position}_{s_1}\} \\
P_3 &= \{\text{at-goal}_{s_2}, \text{position}_{s_2}\} \\
P_4 &= \{\text{at-goal}_{s_1}, \text{position}_{s_1}, \text{position}_p\} \\
P_5 &= \{\text{position}_{s_1}, \text{position}_p\} \\
P_6 &= \{\text{at-goal}_{s_1}, \text{content}_H\} \\
P_7 &= \{\text{at-goal}_{s_1}, \text{content}_E\} \\
P_8 &= \{\text{at-goal}_{s_1}, \text{content}_D\} \\
P_9 &= \{\text{content}_A, \text{content}_E\} \\
P_{10} &= \{\text{at-goal}_{s_1}, \text{content}_Q\}
\end{align*}
\]
(a) Specify the compatibility graph of $C$ and determine its maximal cliques.

(b) Determine the canonical heuristic $h^C$ and simplify it as much as possible.

(c) Not all patterns in $C$ are reasonable. Which can obviously be omitted, and why? What would the canonical heuristic look like if we omitted those patterns before even constructing the compatibility graph?

**Exercise 9.2** (Pattern selection, 5 points)

Consider an SAS$^+$-planning task $\Pi = (V, I, O, \gamma)$. Let $P \subseteq V$ be a pattern such that all variables in $P$ are causally relevant in $P$ and that $P$ is causally connected. Let $v \in V \setminus P$ and $P' = P \cup \{v\}$. Show that the following two statements are equivalent:

(a) All variables in $P'$ are causally relevant in $P'$ and $P'$ is causally connected.

(b) Variable $v$ is a predecessor of some $u \in P$ in $CG(\Pi)$, or $v$ is a successor of some $u \in P$ in $CG(\Pi)$ and is mentioned in $\gamma$.

*Note:* The exercise sheets may and should be worked on in groups of two students. Please state both names on your solution (this also holds for submissions by e-mail).