Exercise 4.1 (A*, 4+2 points)
A knight may move on a chess board with L-shaped moves, jumping 2 grid cells along one axis and one along the other. Consider the 6 × 6 board depicted below, where grey cells can not be entered by the knight. The goal is to move the knight from its initial position in cell (1,1) to cell (5,5) with the smallest possible amount of moves.

(a) Solve the problem with the A* algorithm and the heuristic function

\[ h_1(x_s, y_s, x_g, y_g) = \max\{\left\lceil \frac{|x_s - x_g|}{2} \right\rceil, \left\lceil \frac{|y_s - y_g|}{2} \right\rceil\}, \]

where \((x_s, y_s)\) and \((x_g, y_g)\) are the coordinates of the knight and the goal respectively. In case you encounter two nodes \(\sigma\) and \(\sigma'\) with equal minimal \(f(\sigma) = f(\sigma')\) you may choose the order in which nodes are expanded (i.e., you are allowed to expand the better node first).

(b) Define a heuristic \(h_2\) with the following properties: (a) \(h_1 \leq h_2\) and \(h_2 \nleq h_1\) (i.e., \(h_1(s) \leq h_2(s)\) for all states \(s\), and \(h_1(s) < h_2(s)\) for at least one state \(s\)), (b) \(h_2\) is admissible, and (c) \(h_2\) can be calculated in polynomial time. Show that \(h_2\) indeed has these properties.

Hint: Given two admissible heuristics, it is easy to create a third admissible heuristic that cannot be worse than any of the two original ones by combining them in a suitable way.

Exercise 4.2 (Enforced hill-climbing, 4 points)
Consider the planning task with atoms \(A = \{a, b, c, d, e\}\), initial state \(I = \{a \rightarrow 1, b \rightarrow 1, c \rightarrow 0, d \rightarrow 1, e \rightarrow 0\}\), goal formula \(\gamma = a \land \neg b \land c \land \neg d\) and operators \(O = \{o_1, \ldots, o_5\}\), where

- \(o_1 = \langle a \land b, c \rangle\),
- \(o_2 = \langle a \land c, \neg a \rangle\),
- \(o_3 = \langle a \land b, c \rangle\),
- \(o_4 = \langle \neg a, a \land \neg b \rangle\), and
- \(o_5 = \langle a \land \neg b, \neg d \rangle\).

Solve this problem using the enforced hill-climbing algorithm as introduced in the lecture. Expand nodes by applying operators in the order of their index (the smaller an operator's index the earlier it is applied), and use the goal count heuristic, i.e., the heuristic that estimates the distance from
a given state to the goal by counting how many literals from the goal formula are unsatisfied in that state. Submit the search tree that you obtain and record the solution plan.

Note: The exercise sheets may and should be worked on in groups of two students. Please state both names on your solution (this also holds for submissions by e-mail).