Principles of AI Planning

18. Expressive power

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Motivation
Motivation: Why Analyzing the Expressive Power?

- **Expressive power** is the motivation for designing new planning languages.
- Often there is the question: *Syntactic sugar* or *essential feature*?

→ *Compiling away* or change planning algorithm?

→ If a feature can be compiled away, then it is apparently only *syntactic sugar*.

- Sometimes, however, a compilation can lead to much larger planning domain descriptions or to much longer plans.

→ This means the planning algorithm will probably choke, i.e., it cannot be considered as a compilation.
Example: DNF Preconditions

- Assume we have *DNF preconditions* in STRIPS operators
- This can be compiled away as follows
- Split each operator with a DNF precondition \( c_1 \lor \ldots \lor c_n \) into \( n \) operators with the same effects and \( c_i \) as preconditions
- If there exists a plan for the original planning task there is one for the new planning task and *vice versa*
- The planning task has almost the same size
- The shortest plans have the same size
Example: Conditional effects

- Can we compile away *conditional effects* to STRIPS?
- Example operator: \( \langle a, b \triangleright d \land \neg c \triangleright e \rangle \)
- Can be translated into four operators:
  \( \langle a \land b \land c, d \rangle, \langle a \land b \land \neg c, d \land e \rangle, \ldots \)
- Plan existence and plan size are identical
- *Exponential blowup* of domain description!
- \( \rightarrow \) Can this be avoided?
Propositional STRIPS and Variants
In the following we will only consider propositional STRIPS and some variants of it.

Planning task:

\[ \mathcal{T} = \langle A, I, O, G \rangle. \]

 Often we refer to domain structures \( \mathcal{D} = \langle A, O \rangle. \)
Kambhampati et al [ECP 97] and Gazen & Knoblock [ECP 97]: Disjunctive preconditions are trivial – since they can be translated to basic STRIPS (DNF-preconditions)

Bäckström [AIJ 95]: Disjunctive preconditions are probably essential – since they can not easily be translated to basic STRIPS (CNF-preconditions)

Anderson et al [AIPS 98]: “[D]isjunctive preconditions . . . are . . . essential prerequisites for handling conditional effects” \( \leadsto \) conditional effects imply disjunctive preconditions (?) (General Boolean preconditions)
More “Expressive Power”

\[ \text{STRIPS}_N \] : plain strips with negative literals
\[ \text{STRIPS}_{Bd} \] : precondition in disjunctive normal form
\[ \text{STRIPS}_{Bc} \] : precondition in conjunctive normal form
\[ \text{STRIPS}_B \] : Boolean expressions as preconditions
\[ \text{STRIPS}_C \] : conditional effects
\[ \text{STRIPS}_{C,N} \] : conditional effects & negative literals

...
Theorem

PLANEX is PSPACE-complete for STRIPS$^N$, STRIPS$^{C,B}$, and for all formalisms “between” the two.

Proof.

Follows from theorems proved in the previous lecture.
Expressive Power
Consider **mappings** between planning problems in different formalisms

- that **preserve**
  - solution existence
  - plan size linearly or polynomially etc.
  - the exact plan size
  - the plan “structure”
  - the solutions/plans themselves

- that **are limited**
  - in the size of the result (poly. size)
  - in the *computational resources* (poly. time)

- that **transform**
  - entire planning instances
  - domain structure and states in isolation
Measuring Expressive Power

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When measuring the expressiveness of planning formalisms, domain structures should be considered independently from states.
The Right Method: Compilation Schemes (Simplified)

- Transform domain structure \( D = \langle A, O \rangle \) (with polynomial blowup) to \( D' \) preserving solution existence
- Only trivial changes to states (independent of operator set)
- Resulting plans \( \pi' \) should not grow too much (additive constant, linear growth, polynomial growth)

\[ \Rightarrow \] Similar to knowledge compilation, with operators as the fixed part and initial states & goals as the varying part
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\( \leadsto \) Similar to knowledge compilation, with operators as the fixed part and initial states & goals as the varying part
Y ≤ X (Y is compilable to X) iff there exists a compilation scheme from Y to X.

- Y ≤₁ X: preserving plan size exactly (modulo additive constants)
- Y ≤ₖ X: preserving plan size linearly (in |π|)
- Y ≤ₚ X: preserving plan size polynomially (in |π| and |D|)
- Y ≤ₓₚ X: polynomial-time compilability

Theorem
For all x, y, the relations ≤ₓₚ y are transitive and reflexive.
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Y≤^c X: preserving plan size linearly (in |π|)
Y≤^p X: preserving plan size polynomially (in |π| and |D|)
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Theorem

For all x, y, the relations ≤^x y are transitive and reflexive.
Back-Translatability

- Shouldn’t we also require that plans in the compiled instance can be translated back to the original formalism?
- Yes, if we want to use this technique, one should require that!
- In all positive cases, there was never any problem to translate the plan back
- For the negative case, it is easier to prove non-existence
- So, in order to prove negative results, we do not need it, for positive it never had been a problem

⇝ So, similarly to the concentration on decision problems when determining complexity, we simplify things here
A (Trivial) Positive Result: $\text{STRIPS}_{Bd} \preceq_1^p \text{STRIPS}_N$

DNF preconditions can be "compiled away."

Assume operator $o = \langle c, e \rangle$ and

$$c = L_1 \lor \ldots \lor L_k$$

with $L_i$ being a conjunction of literals. Create $k$ operators $o_i = \langle L_i, e \rangle$

1. compilation is solution-preserving,
2. $D'$ is only polynomially larger than $D$,
3. compilation can be computed in polynomial time,
4. resulting plans do not grow at all.

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Another Positive Result: \[ \text{STRIPS}^C_{Bc} \preceq_p^C \text{STRIPS}^C_{C,N} \]

CNF preconditions can be “compiled away” – provided we have already conditional effects.

- Evaluate the truth value of all disjunctions appearing in operators by using a special evaluation operator with conditional effects that make new “clause atoms” true.
- Alternate between executing original operators (clauses replaced by new atoms) and evaluation operators.

\[ \Rightarrow \] Operator sets grow only \textit{polynomially}.

\[ \Rightarrow \] Plans are double as long as the original plans.

\[ \Rightarrow \] Anderson et al.’s conjecture holds in a weak version.
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A First Negative Result: Conditional Effects Cannot be Compiled into Boolean Preconditions

Consider domain $\mathcal{D}$ with only one (STRIPS$_{C,B}$) operator $o$:

$$\langle \top, (p_1 \triangleright \neg p_1) \land (\neg p_1 \triangleright p_1) \land \ldots \land (p_k \triangleright \neg p_k) \land (\neg p_k \triangleright p_k) \rangle,$$

which “inverts” a given state. For all $(I, G)$ with

$$G = \bigwedge \{ \neg v \mid v \in A, I \models v \} \land \bigwedge \{ v \mid v \in A, I \not\models v \},$$

there exists a STRIPS$_{C,B}$ one-step plan.

Assume there exists a compilation preserving plan size linearly leading to a STRIPS$_B$ domain structure $\mathcal{D}'$. There are exponentially many possible initial states, but only polynomially many different $c$-step plans for $\mathcal{D}'$. Some STRIPS$_B$ plan $\pi$ is used for different initial states $I_1$, $I_2$ (for large enough $k$). Let $v$ be a variable with $I_1(v) \neq I_2(v)$.

$\Rightarrow$ In one case, $v$ must be set by $\pi$, in the other case, it must be cleared.

$\Rightarrow$ This is not possible in an unconditional plan.

$\Rightarrow$ The transformation is **not solution preserving**!

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$\Rightarrow$ In one case, $v$ must be set by $\pi$, in the other case, it must be cleared.
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$\Rightarrow$ The transformation is not solution preserving!
$\Rightarrow$ Conditional effects cannot be compiled away (if plan size can grow only linearly)
Another Negative Result: $\text{STRIPS}_{Bc} \not\leq^c \text{STRIPS}_N$

$k$-FISEX: Planning problem with fixed plan length $k$ and varying initial state. Does there exist an initial state leading to a successful $k$-step plan?

1-FISEX is NP-complete for $\text{STRIPS}_{Bc}$ (= SAT).

$k$-FISEX is polynomial for $\text{STRIPS}_N$ (regression analysis)

$$\implies \text{STRIPS}_{Bc} \not\leq_p \text{STRIPS}_N \text{ (if } P \neq \text{NP)}$$

Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

$$\implies \text{Bäckström’s conjecture holds} \text{ in the compilation framework.}$$
Another Negative Result: \(\text{STRIPS}_{Bc} \not\leq^c \text{STRIPS}_N\)

\(k\)-FISEX: Planning problem with fixed plan length \(k\) and varying initial state. Does there exist an initial state leading to a successful \(k\)-step plan?

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\(\leadsto \text{STRIPS}_{Bc} \not\leq^c_p \text{STRIPS}_N\) (if \(\text{P} \neq \text{NP}\))

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\(\leadsto \text{Bäckström’s conjecture holds}\) in the compilation framework.
Another Negative Result: \( \text{STRIPS}_{Bc} \not\preceq^c \text{STRIPS}_N \)

**k-FISEX:** Planning problem with fixed plan length \( k \) and varying initial state. Does there exist an initial state leading to a successful \( k \)-step plan?

1. FISEX is NP-complete for \( \text{STRIPS}_{Bc} (= \text{SAT}) \).
2. \( k \)-FISEX is polynomial for \( \text{STRIPS}_N \) (regression analysis)

\[ \leadsto \text{STRIPS}_{Bc} \not\preceq^c \text{STRIPS}_N \text{ (if } P \neq \text{NP)} \]

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Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

$\implies$ Bäckström’s conjecture holds in the compilation framework.
Another Negative Result: $\text{STRIPS}_{Bc} \not\preceq_c \text{STRIPS}_N$

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Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

$\iff \text{Bäckström’s conjecture holds}$ in the compilation framework.
Another Negative Result: $\text{STRIPS}_{Bc} \not\leq^c \text{STRIPS}_N$

$k$-FISEX: Planning problem with fixed plan length $k$ and varying initial state. Does there exist an initial state leading to a successful $k$-step plan?

1-FISEX is NP-complete for $\text{STRIPS}_{Bc}$ (≡ SAT).

$k$-FISEX is polynomial for $\text{STRIPS}_N$ (regression analysis)

$$\leadsto \text{STRIPS}_{Bc} \not\leq^c_p \text{STRIPS}_N \text{ (if } P \neq \text{NP)}$$

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Using a technique first used by Kautz & Selman, one can show that even arbitrary compilations can be ruled out – provided the polynomial hierarchy does not collapse. The proof method uses non-uniform complexity classes such as $P/poly$.

\[ \iff \text{Bäckström’s conjecture holds} \text{ in the compilation framework.} \]
Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) (\(= \text{NC}^1\))

- Conditional effects can simulate only families of circuits with fixed depth (\(= \text{AC}^0\)).

- The parity function can be expressed in the first framework (\(\text{NC}^1\)) while it cannot be expressed in the second (\(\text{AC}^0\)).

\(\rightarrow\) The negative result follows unconditionally!
Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) ($\equiv \text{NC}^1$)

Conditional effects can simulate only families of circuits with fixed depth ($\equiv \text{AC}^0$).

The parity function can be expressed in the first framework ($\text{NC}^1$) while it cannot be expressed in the second ($\text{AC}^0$).

The negative result follows unconditionally!
A Final Negative Result: Boolean Preconditions Cannot be Compiled Away Even in the Presence of Conditional Effects

- Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) (= NC\textsuperscript{1})
- Conditional effects can simulate only families of circuits with fixed depth (= AC\textsuperscript{0}).
- The parity function can be expressed in the first framework (NC\textsuperscript{1}) while it cannot be expressed in the second (AC\textsuperscript{0}).

⇝ The negative result follows unconditionally!
Boolean preconditions have the power of families of Boolean circuits with logarithmic depth (because Boolean formula have this power) \( (= \text{NC}^1) \)

Conditional effects can simulate only families of circuits with fixed depth \( (= \text{AC}^0) \).

The parity function can be expressed in the first framework \( (\text{NC}^1) \) while it cannot be expressed in the second \( (\text{AC}^0) \).

\( \Rightarrow \) The negative result follows unconditionally!
We know what Boolean circuits are (directed, acyclic graphs with different types of nodes: and, or, not, input, output)

- **Size of circuit** = number of gates
- **Depth of circuit** = length of longest path from input gate to output gate

When we want to recognize formal languages with circuits, we need a sequence of circuits with an increasing number of input gates \( \mapsto \) family of circuits

- Families with polynomial size and poly-log \( (\log^k n) \) depth
- Complexity classes \( \text{NC}^k \) (Nick’s class)

\[ \text{NC} = \bigcup_k \text{NC}^k \subseteq P \], the class of problems that can be solved efficiently in parallel

- The class of languages that can be characterized by polynomially sized Boolean formulae is identical to \( \text{NC}^1 \)
The classes $\text{AC}^k$

- The classes $\text{NC}^k$ are defined with a fixed fan-in
- If we have unbounded fan-in, we get the classes $\text{AC}^k$
  - gate types: NOT, $n$-ary AND, $n$-ary OR for all $n \geq 2$
- Obviously: $\text{NC}^k \subseteq \text{AC}^k$
- Possible to show: $\text{AC}^{k-1} \subseteq \text{NC}^k$
- The parity language is in $\text{NC}^1$, but not in $\text{AC}^0$!
Accepting languages with families of domain structures with fixed goals

- We will view families of domain structures with fixed goals and fixed size plans as “machines” that accept languages.
- Consider families of poly-sized domain structures in STRIPS$_B$ and use one-step plans for acceptance.
- Obviously, this is the same as using Boolean formulae.
- All languages in NC$^1$ can be accepted in this way.
Simulating STRIPS\(_{\text{C},N}\) c-step Plans with AC\(^0\) circuits (1)

- Represent each operator and then chain the actions together (\(O(|O|^c)\) different plans):
Simulating STRIPS$_{C,N}$ $c$-Step Plans with AC$^0$ circuits (2)

- For each single action (precondition testing (a), conditional effects (b), and the computation of effects (c))
Theorem

\[ \text{STRIPS}_B \not\leq^c \text{STRIPS}_{C,N}. \]

Proof.

Assuming \( \text{STRIPS}_B \leq^c \text{STRIPS}_{C,N} \) has the consequence that the underlying compilation scheme could be used to compile a \( \text{NC}^1 \) circuit family into an \( \text{AC}^0 \) circuit family, which is impossible in the general case.
All other potential positive results have been ruled out by our 3 negative results and transitivity.
Summary
Compilation schemes seem to be the right method to measure the relative expressive power of planning formalisms.

Either we get a positive result preserving plan size linearly with a polynomial-time compilation.

Or we get an impossibility result.

→ Results are relevant for building planning systems.

⇝ CNF preconditions do not add much when we have already conditional effects.

Note: In all cases we can get a positive result if we allow for a polynomial blow-up of the plans.