Principles of AI Planning

6. Planning as search: search algorithms

Bernhard Nebel and Robert Mattmüller

Albert-Ludwigs-Universität Freiburg

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6.1 Introduction to search algorithms for planning

6.2 Uninformed search algorithms

6.3 Heuristic search algorithms
6.1 Introduction to search algorithms for planning

- Search nodes & search states
- Search for planning
- Common procedures for search algorithms
Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   ⇝ previous chapter
2. search space representation: states/sets of states
   ⇝ previous chapter
3. search algorithm: uninformed/heuristic; systematic/local
   ⇝ this chapter
4. search control: heuristics, pruning techniques
   ⇝ next chapters
Search

- Search algorithms are used to find solutions (plans) for transition systems in general, not just for planning tasks.
- Planning is one application of search among many.
- In this chapter, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.
- Most of this is review of material that should be known (details: Russell and Norvig’s textbook).
Search states vs. search nodes

In search, one distinguishes:

- **search states** $s \leadsto$ states (vertices) of the transition system
- **search nodes** $\sigma \leadsto$ search states plus information on where/when/how they are encountered during search

What is in a search node?

Different search algorithms store different information in a search node $\sigma$, but typical information includes:

- $\text{state}(\sigma)$: associated search state
- $\text{parent}(\sigma)$: pointer to search node from which $\sigma$ is reached
- $\text{action}(\sigma)$: an action/operator leading from $\text{state}(\text{parent}(\sigma))$ to $\text{state}(\sigma)$
- $g(\sigma)$: cost of $\sigma$ (length of path from the root node)

For the root node, $\text{parent}(\sigma)$ and $\text{action}(\sigma)$ are undefined.
Search states vs. planning states

Search states ≠ (planning) states:

- Search states don’t have to correspond to states in the planning sense.
  - progression: search states ≈ (planning) states
  - regression: search states ≈ sets of states (formulae)

- Search algorithms for planning where search states are planning states are called state-space search algorithms.

- Strictly speaking, regression is not an example of state-space search, although the term is often used loosely.

- However, we will put the emphasis on progression, which is almost always state-space search.
Required ingredients for search

A general search algorithm can be applied to any transition system for which we can define the following three operations:

- **init()**: generate the initial state
- **is-goal(s)**: test if a given state is a goal state
- **succ(s)**: generate the set of successor states of state $s$, along with the operators through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a search space (a very similar notion to a transition system).
Search for planning: progression

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

**Search space for progression search**

**states:** all states of $\Pi$ (assignments to $A$)

- $\text{init}(\cdot) = I$
- $\text{is-goal}(s) = \begin{cases} \text{true} & \text{if } s \models \gamma \\ \text{false} & \text{otherwise} \end{cases}$
- $\text{succ}(s) = \{ \langle o, s' \rangle \mid o \in O, s' = \text{app}_o(s) \}$
Search for planning: regression

Let $\Pi = \langle A, I, O, \gamma \rangle$ be a planning task.

Search space for regression search

states: all formulae over $A$ (how many?)

- init() = $\gamma$
- $\text{is-goal}(\varphi) = \begin{cases} \text{true} & \text{if } I \models \varphi \\ \text{false} & \text{otherwise} \end{cases}$
- $\text{succ}(\varphi) = \{ \langle o, \varphi' \rangle \mid o \in O, \varphi' = \text{regr}_o(\varphi), \varphi' \text{ is satisfiable} \}$
  (modified if splitting is used)
Classification of search algorithms

uninformed search vs. heuristic search:
- uninformed search algorithms only use the basic ingredients for general search algorithms
- heuristic search algorithms additionally use heuristic functions which estimate how close a node is to the goal

systematic search vs. local search:
- systematic algorithms consider a large number of search nodes simultaneously
- local search algorithms work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are crossbreeds (e.g., enforced hill-climbing)
Classification: what works where in planning?

uninformed vs. heuristic search:

- For **satisficing** planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For **optimal** planning, the difference is less pronounced.

systematic search vs. local search:

- For **satisficing** planning, the most successful algorithms are somewhere between the two extremes.
- For **optimal** planning, systematic algorithms are required.
Common procedures for search algorithms

Before we describe the different search algorithms, we introduce three procedures used by all of them:

- **make-root-node**: Create a search node without parent.
- **make-node**: Create a search node for a state generated as the successor of another state.
- **extract-solution**: Extract a solution from a search node representing a goal state.
Procedure make-root-node

**make-root-node**: Create a search node without parent.

```python
def make-root-node(s):
    σ := new node
    state(σ) := s
    parent(σ) := undefined
    action(σ) := undefined
    g(σ) := 0
    return σ
```
Procedure make-node

**make-node**: Create a search node for a state generated as the successor of another state.

```python
def make-node(σ, o, s):
    σ′ := new node
    state(σ′) := s
    parent(σ′) := σ
    action(σ′) := o
    g(σ′) := g(σ) + 1
    return σ′
```
Procedure extract-solution

**extract-solution**: Extract a solution from a search node representing a goal state.

```python
def extract_solution(σ):
    solution := new list
    while parent(σ) is defined:
        solution.push-front(action(σ))
        σ := parent(σ)
    return solution
```
6.2 Uninformed search algorithms

- Breadth-first search without duplicate detection
- Breadth-first search with duplicate detection
- Random walk
Uninformed search algorithms

- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

Popular uninformed systematic search algorithms:

- breadth-first search
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:

- random walk
Breadth-first search without duplicate detection

Breadth-first search

\[ queue := \textbf{new} \ \text{fifo-queue} \]
\[ queue.\text{push-back}(\text{make-root-node}(\text{init}())) \]

\[ \text{while not } queue.\text{empty}(): \]
\[ \quad \sigma = queue.\text{pop-front}() \]
\[ \quad \text{if is-goal(state(} \sigma \text{)):} \]
\[ \qquad \text{return} \ \text{extract-solution}(\sigma) \]
\[ \quad \text{for each } \langle o, s \rangle \in \text{succ(state}(\sigma)\text{)}: \]
\[ \qquad \sigma' := \text{make-node}(\sigma, o, s) \]
\[ \qquad queue.\text{push-back}(\sigma') \]

\[ \text{return } \text{unsolvable} \]

- Possible improvement: \textbf{duplicate detection} (see next slide).
- Another possible improvement: test if \( \sigma' \) is a goal node; if so, terminate immediately. (We don’t do this because it obscures the similarity to some of the later algorithms.)
Breadth-first search with duplicate detection

queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed := ∅
while not queue.empty():
    σ = queue.pop-front()
    if state(σ) ∉ closed:
        closed := closed ∪ {state(σ)}
    if is-goal(state(σ)):
        return extract-solution(σ)
    for each ⟨o, s⟩ ∈ succ(state(σ)):
        σ′ := make-node(σ, o, s)
        queue.push-back(σ′)
return unsolvable


Breadth-first search with duplicate detection

\[
\begin{align*}
\text{queue} & := \textbf{new} \ \text{fifo-queue} \\
\text{queue} & .\text{push-back}(\text{make-root-node}(\text{init}())) \\
\text{closed} & := \emptyset \\
\textbf{while} \ \text{not} \ \text{queue}.\text{empty}(): \\
& \quad \sigma = \text{queue}.\text{pop-front}() \\
& \quad \textbf{if} \ \text{state}(\sigma) \notin \text{closed}: \\
& \quad \quad \text{closed} := \text{closed} \cup \{\text{state}(\sigma)\} \\
& \quad \quad \textbf{if} \ \text{is-goal}(\text{state}(\sigma)): \\
& \quad \quad \quad \textbf{return} \ \text{extract-solution}(\sigma) \\
& \quad \quad \textbf{for each} \ \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)):\ \\
& \quad \quad \quad \sigma' := \text{make-node}(\sigma, o, s) \\
& \quad \quad \quad \text{queue}.\text{push-back}(\sigma') \\
\textbf{return} \ \text{unsolvable}
\end{align*}
\]
Random walk

Random walk

$\sigma := \text{make-root-node}(\text{init}())$

\textbf{forever}:

\hspace{1em} \textbf{if} is-goal(state($\sigma$)):

\hspace{2em} \textbf{return} extract-solution($\sigma$)

Choose a random element $\langle o, s \rangle$ from $\text{succ}(\text{state}(\sigma))$.

$\sigma := \text{make-node}(\sigma, o, s)$

- The algorithm usually does not find any solutions, unless almost every sequence of actions is a plan.
- Often, it runs indefinitely without making progress.
- It can also fail by reaching a \textbf{dead end}, a state with no successors. This is a weakness of many local search approaches.
6.3 Heuristic search algorithms

- Heuristics: definition and properties
- Systematic heuristic search algorithms
- Heuristic local search algorithms
Heuristic search algorithms: systematic

- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular systematic heuristic search algorithms:
- greedy best-first search
- $A^*$
- weighted $A^*$
- $IDA^*$
- depth-first branch-and-bound search
- ...
Heuristic search algorithms: local

- Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic local search algorithms:
- hill-climbing
- enforced hill-climbing
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- ...
Heuristic search: idea
Required ingredients for heuristic search

A **heuristic search algorithm** requires one more operation in addition to the definition of a search space.

**Definition (heuristic function)**

Let $\Sigma$ be the set of nodes of a given search space. A **heuristic function** or **heuristic** (for that search space) is a function $h : \Sigma \rightarrow \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the **heuristic estimate** or **heuristic value** of heuristic $h$ for node $\sigma$. It is supposed to estimate the distance from $\sigma$ to the nearest goal node.
What exactly is a heuristic estimate?

What does it mean that $h$ “estimates the goal distance”?

- For most heuristic search algorithms, $h$ does not need to have any strong properties for the algorithm to work (= be correct and complete).

- However, the **efficiency** of the algorithm closely relates to how accurately $h$ reflects the actual goal distance.

- For some algorithms, like $A^*$, we can prove strong formal relationships between properties of $h$ and properties of the algorithm (optimality, dominance, run-time for bounded error, ...)

- For other search algorithms, “it works well in practice” is often as good an analysis as one gets.
Heuristics applied to nodes or states?

- Most texts apply heuristic functions to states, not nodes.
- This is slightly less general than our definition:
  - Given a state heuristic \( h \), we can define an equivalent node heuristic as \( h'(\sigma) := h(state(\sigma)) \).
  - The opposite is not possible. (Why not?)
- There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on how we ended up in a given state \( s \)?
- We call heuristics which don’t just depend on \( state(\sigma) \) pseudo-heuristics.
- In practice there are sometimes good reasons to have the heuristic value depend on the generating path of \( \sigma \) (e.g., the landmark pseudo-heuristic, Richter et al. 2008).
Perfect heuristic

Let $\Sigma$ be the set of nodes of a given search space.

**Definition (optimal/perfect heuristic)**

The optimal or perfect heuristic of a search space is the heuristic $h^*$ which maps each search node $\sigma$ to the length of a shortest path from $\text{state}(\sigma)$ to any goal state.

**Note:** $h^*(\sigma) = \infty$ iff no goal state is reachable from $\sigma$. 
Properties of heuristics

A heuristic $h$ is called

- **safe** if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- **goal-aware** if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- **admissible** if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- **consistent** if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that $\sigma'$ is a successor of $\sigma$

Relationships?
Greedy best-first search

Greedy best-first search (with duplicate detection)

\[ \text{open} := \textbf{new} \text{ min-heap ordered by } (\sigma \mapsto h(\sigma)) \]
\[ \text{open}.\text{insert}(\text{make-root-node}(\text{init}())) \]
\[ \text{closed} := \emptyset \]

\textbf{while not} open.\text{empty}():

\[ \sigma = \text{open}.\text{pop-min}() \]

\textbf{if} state(\sigma) \notin \text{closed}:

\[ \text{closed} := \text{closed} \cup \{\text{state}(\sigma)\} \]

\textbf{if} is-goal(state(\sigma)):

\[ \text{return} \ \text{extract-solution}(\sigma) \]

\textbf{for each} \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)):

\[ \sigma' := \text{make-node}(\sigma, o, s) \]

\textbf{if} \ h(\sigma') < \infty:

\[ \text{open}.\text{insert}(\sigma') \]

\textbf{return} unsolvable
Properties of greedy best-first search

- one of the three most commonly used algorithms for satisficing planning
- **complete** for safe heuristics (due to duplicate detection)
- **suboptimal** unless $h$ satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of $h$ (e.g., scaling with a positive constant or adding a constant)
A* (with duplicate detection and reopening)

\[ \text{open} := \text{new min-heap ordered by } (\sigma \mapsto g(\sigma) + h(\sigma)) \]
\[ \text{open}.\text{insert}(\text{make-root-node}(\text{init}())) \]

\[ \text{closed} := \emptyset \]
\[ \text{distance} := \emptyset \]
\[ \text{while not } \text{open}.\text{empty}(): \]
\[ \quad \sigma = \text{open}.\text{pop-min}() \]
\[ \quad \text{if } \text{state}(\sigma) \notin \text{closed} \text{ or } g(\sigma) < \text{distance}(\text{state}(\sigma)): \]
\[ \quad \quad \text{closed} := \text{closed} \cup \{ \text{state}(\sigma) \} \]
\[ \quad \quad \text{distance}(\text{state}(\sigma)) := g(\sigma) \]
\[ \quad \text{if } \text{is-goal}(\text{state}(\sigma)): \]
\[ \quad \quad \text{return } \text{extract-solution}(\sigma) \]
\[ \quad \text{for each } \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)): \]
\[ \quad \quad \sigma' := \text{make-node}(\sigma, o, s) \]
\[ \quad \quad \text{if } h(\sigma') < \infty: \]
\[ \quad \quad \quad \text{open}.\text{insert}(\sigma') \]
\[ \text{return unsolvable} \]
A* example

Example

Heuristic search  Systematic search
A* example

Example
A* example

Example
A* example

Example
A* example
Terminology for A*

- **f value of a node**: defined by $f(\sigma) := g(\sigma) + h(\sigma)$
- **generated nodes**: nodes inserted into open at some point
- **expanded nodes**: nodes $\sigma$ popped from open for which the test against closed and distance succeeds
- **reexpanded nodes**: expanded nodes for which $\text{state}(\sigma) \in \text{closed}$ upon expansion (also called reopened nodes)
Properties of A*

- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- complete for safe heuristics (even without duplicate detection)
- optimal if $h$ is admissible (even without duplicate detection)
- never reopens nodes if $h$ is consistent

Implementation notes:

- in the heap-ordering procedure, it is considered a good idea to break ties in favour of lower $h$ values
- can simplify algorithm if we know that we only have to deal with consistent heuristics
- common, hard to spot bug: test membership in closed at the wrong time
Weighted A*

Weighted A* (with duplicate detection and reopening)

\[
\begin{align*}
open & := \textbf{new} \text{ min-heap ordered by } (\sigma \mapsto g(\sigma) + W \cdot h(\sigma)) \\
open.\text{insert}(\text{make-root-node(init())})
\end{align*}
\]

\[
\begin{align*}
closed & := \emptyset \\
distance & := \emptyset \\
\textbf{while not} & \open.\text{empty}(): \\
\quad \sigma & = \open.\text{pop-min}() \\
\quad \textbf{if} & \ \text{state}(\sigma) \notin \closed \textbf{ or } g(\sigma) < \text{distance(state}(\sigma)):
\quad \quad \closed & := \closed \cup \{\text{state}(\sigma)\} \\
\quad \quad \text{distance}(\sigma) & := \text{g}(\sigma) \\
\quad \textbf{if} & \ \text{is-goal(state}(\sigma)):
\quad \quad \textbf{return} \ \text{extract-solution}(\sigma)
\quad \textbf{for each} & \langle o, s \rangle \in \text{succ(state}(\sigma)):
\quad \quad \sigma' & := \text{make-node}(\sigma, o, s) \\
\quad \quad \textbf{if} & \ h(\sigma') < \infty:
\quad \quad \quad \open.\text{insert}(\sigma')
\quad \textbf{return} \ \text{unsolvable}
\end{align*}
\]
Properties of weighted A* 

The weight $W \in \mathbb{R}_0^+$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like A*
- for $W \rightarrow \infty$, behaves like greedy best-first search

Properties:
- one of the most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to A*, replacing optimal with bounded suboptimal: generated solutions are at most a factor $W$ as long as optimal ones
Hill-climbing

\[
\begin{align*}
\sigma &:= \text{make-root-node}(\text{init}()) \\
\text{forever:} & \\
\text{if } \text{is-goal}(\text{state}(\sigma)) : & \\
\quad & \text{return } \text{extract-solution}(\sigma) \\
\Sigma' &:= \{ \text{make-node}(\sigma, o, s) | \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)) \} \\
\sigma &:= \text{an element of } \Sigma' \text{ minimizing } h \text{ (random tie breaking)}
\end{align*}
\]

- can easily get stuck in local minima where immediate improvements of \( h(\sigma) \) are not possible
- many variations: tie-breaking strategies, restarts
Enforced hill-climbing

Enforced hill-climbing: procedure improve

```python
def improve(σ₀):
    queue := new fifo-queue
    queue.push-back(σ₀)
    closed := ∅
    while not queue.empty():
        σ = queue.pop-front()
        if state(σ) ∉ closed:
            closed := closed ∪ {state(σ)}
            if h(σ) < h(σ₀):
                return σ
        for each ⟨o, s⟩ ∈ succ(state(σ)):
            σ′ := make-node(σ, o, s)
            queue.push-back(σ′)
    fail

⇝ breadth-first search for more promising node than σ₀
```
Enforced hill-climbing (ctd.)

Enforced hill-climbing

\[ \sigma := \text{make-root-node}(\text{init}()) \]

\textbf{while not} is-goal(state(\sigma)):

\[ \sigma := \text{improve}(\sigma) \]

\textbf{return} extract-solution(\sigma)

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from \( \sigma_0 \))
- complete for undirected search spaces (where the successor relation is symmetric) if \( h(\sigma) = 0 \) for all goal nodes and only for goal nodes
Summary

- distinguish: planning states, search states, search nodes
  - planning state: situation in the world modelled by the task
  - search state: subproblem remaining to be solved
    - In state-space search (usually progression search), planning states and search states are identical.
    - In regression search, search states usually describe sets of states (“subgoals”).
  - search node: search state + info on “how we got there”

- search algorithms mainly differ in order of node expansion
  - uninformed vs. informed (heuristic) search
  - local vs. systematic search
Summary (ctd.)

- **heuristics**: estimators for “distance to goal node”
  - usually: the more accurate, the better performance
  - desiderata: safe, goal-aware, admissible, consistent
  - the ideal: perfect heuristic $h^*$

- most common algorithms for **satisficing planning**:
  - greedy best-first search
  - weighted A*
  - enforced hill-climbing

- most common algorithm for **optimal planning**:
  - A*