Exercise 2.1 (Deterministic Finite Automata)

Construct DFAs that recognize the following languages. In all cases, $\Sigma = \{0, 1\}$.

- The empty language $\emptyset$.
- Only $\epsilon$.
- All the languages.
- The set of strings with three consecutive 0’s (not necessarily at the end).
- The set of strings that does not contain 00.
- The set of strings that either begin or end with the same number twice.
- The set of strings such that each block of five consecutive symbols contains at least two 1’s.
- The set of strings such that the number of 0’s is divisible by 5, and the number of 1’s is divisible by 3.
- The set of strings beginning with a 1 that, when interpreted as a binary integer, is a multiple of 5. For example, strings 101, 1010, 1111 are in the language and 0, 100, 111 are not.

Exercise 2.2 (Non Deterministic Finite Automata)

Construct NFAs that recognize the following languages.

- The set of strings where letters appear in reversed alphabetical order. $\Sigma = \{a, b, c\}$.
- The set of strings that contains an $a$ in the odd positions. $\Sigma = \{a, b, c\}$.
- The set of strings that contains at least two occurrences of $cb$ and ends in $bb$. $\Sigma = \{a, b, c\}$
- The set of strings where the 5th symbol from the end is an $a$. $\Sigma = \{a, b\}$
- The set of strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has appeared before.
- The set of strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has not appeared before.
- The set of strings such that there are no two $a$’s separated by a number of positions that is a multiple of 4. Note that 0 is an allowable multiple of 4.
Exercise 2.3 (Regular Expressions)

Write regular expressions for the following languages:

- The set of strings over alphabet $\Sigma = \{a, b, c\}$ containing at least one $a$ and at least one $b$.
- The set of strings over alphabet $\Sigma = \{a, b, c\}$ where every $a$ is followed immediately by a $c$.
- The set of strings over alphabet $\Sigma = \{a, b, c\}$ that have an even number of the substring $ac$.
- The set of strings of 0’s and 1’s whose fifth symbol from the right end is a 1.
- The set of strings of 0’s and 1’s with at most one pair of consecutive 1’s.
- The set of strings of 0’s and 1’s such that every pair of adjacent 0’s appears before any pair of adjacent 1’s.
- The set of strings of 0’s and 1’s whose number of 0’s is divisible by five.
- The set of strings of 0’s and 1’s not containing 101 as a substring.
- The phone numbers in Germany.

Exercise 2.4 (Regular Expressions Properties)

Prove or disprove each of the following statements about regular expressions

- $r + s = s + r$
- $(r^*)^* = r^*$
- $(r + s)^* = r^* + s^*$
- $(rs + r)^*rs = (rr^*s)^*$

Exercise 2.5 (The Pumping Lemma)

Prove that the following are not regular languages by using the pumping lemma.

- $L = \{\text{All strings with an equal number of 0s and 1s not in any particular order.}\}$
- $L = \{0^n110^n \mid n \geq 1\}$
- $L = \{0^n1^m0^n \mid n, m \in \mathbb{N}\}$
- $L = \{0^n1^m \mid n, m \in \mathbb{N} \text{ such that } n \leq m\}$
- $L = \{0^n12^n \mid n \geq 1\}$
- $L = \{0^n \mid n \text{ is a perfect cube}\}$
- $L = \{0^n \mid n \text{ is a power of 2}\}$
- The language of palindromes.
- The set of strings of 0’s and 1’s that are of the form $ww$, that is, some string repeated.
- The set of strings of 0’s and 1’s that are of the form $ww^R$, that is, some string followed by its reverse.
- The set of strings of 0’s and 1’s of the form $w\overline{w}$, where $\overline{w}$ is formed from $w$ by replacing all 0’s by 1’s and vice versa; e.g., $0110 = 100$ and 011100 is an example of a string in the language.