Exercise 14.1

(P, 1.5 + 0.5 marks)

(a) Show that $P$ is closed under union, complement, and concatenation.

(b) The complexity class $coP$ contains all languages $L$ whose complement is in $P$. Formally, $coP = \{ L | \overline{L} \in P \}$. Is $P = coP$?

Exercise 14.2

(Post’s Correspondence Problem, 2 + 2 marks)

An instance of Post’s Correspondence Problem (PCP) consists of two lists of strings over some alphabet $\Sigma$; the two lists must be of equal length, and we generally refer to them as $A$ and $B$. We write $A = w_1, w_2, \ldots, w_k$ and $B = x_1, x_2, \ldots, x_k$, for some integer $k$. For each $i$, the pair $(w_i, x_i)$ is said to be a corresponding pair.

We say this instance of PCP has a solution, if there is a sequence of one or more integers $i_1, i_2, \ldots, i_m$ that, when interpreted as indexes for strings in the $A$ and $B$ lists, yield the same string. That is, $w_{i_1}w_{i_2}\ldots w_{i_m} = x_{i_1}x_{i_2}\ldots x_{i_m}$. We say that the sequence $i_1, i_2, \ldots, i_m$ is a solution to this instance of PCP.

Consider the following instances $Y$ of PCP. Which instance $Y$ has a solution? Justify your answer.

a) $Y = \{ (aba, a), (ba, babab) \}$

b) $Y = \{ (bab, ba), (aaabb, a), (ab, ababb) \}$

Exercise 14.3

(Reduction, 1 + 3 marks)

A $k$-clique in a graph $G$ is a set of $k$ nodes of $G$ such that there is an edge between every two nodes in the clique. Thus, a 2−clique is just a pair of nodes connected by an edge, and a 3−clique is a triangle. Consider the following problems.

$CLIQUE = \{ (G, k) | G$ is an undirected graph that contains a clique with $k$ vertices $\}$

$NODE COVER = \{ (G, k) | G$ is an undirected graph that has a node cover with $k$ or fewer nodes. $\}$

(a) How many edges does a $k$−clique have, as a function of $k$?

(b) Prove that CLIQUE is $NP$−complete by reducing the node-cover problem to CLIQUE.