Exercise 12.1 (Decidable Languages, 1 + 1 marks)
Let $L$ and $L'$ be decidable languages. Prove the following properties.

(a) The complement $\overline{L}$ is decidable.

(b) The union $L \cup L'$ is decidable.

Exercise 12.2 (Decidable Languages, 1.5 + 1 + 1.5 marks)
Show that the following languages are decidable:

(a) $EQ_{DFA,RE} = \{ \langle D, R \rangle \mid D$ is a DFA and $R$ is a regular expression and $L(D) = L(R) \}$

(b) $A_{CFG} = \{ \langle G \rangle \mid G$ is a CFG that generates $\epsilon \}$

(c) $ALL_{DFA} = \{ \langle A \rangle \mid A$ is a DFA that recognizes $\Sigma^* \}$

Exercise 12.3 (Undecidable Languages, 4 marks)
Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

$$N = \{ \langle M, w \rangle \mid M$ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on $w \}.$$

Show that $N$ is undecidable. Hint: Use a reduction from $A_{TM}$. 