Exercise 7.1 (Regular expressions, 0.5 + 0.5 + 0.5 + 0.5 marks)
Consider the following regular expressions. What language do they recognize? Give two strings that are members of the corresponding language and two strings which are not members – a total of four strings for each part. Assume the alphabet $\Sigma = \{a, b, c\}$ in all parts.

(a) $a^*b^*c^*$

(b) $\Sigma^*aba\Sigma^*$

(c) $((a \cup c)^*b(a \cup c)^*b(a \cup c)^*b(a \cup c)^*)^*$

(d) $b \cup b\Sigma^*b$

Exercise 7.2 (Regular Expressions, 0.5 + 0.5 + 1 + 1 + 1 marks)
Construct regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$

(a) $L_1 = \{w \in \Sigma^* \mid$ every 0 in $w$ is immediately followed by a 1\}$

(b) $L_2 = \{w \in \Sigma^* \mid$ the second or the third position from the end is a 1\}$

(c) $L_3 = \{w \in \Sigma^* \mid w$ consists of alternating 0 and 1\}$

(d) $L_4 = \{w \in \Sigma^* \mid w$ does not contain 11\}$

(e) $L_5 = \{w \in \Sigma^* \mid w$ ends in even number of 0s\}$

Exercise 7.3 (NFAs and Regular Expressions, 2.5 + 1.5 marks)
Consider the regular expression $(30 \cup 75 \cup 45)^* \circ 10$ (over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$).

(a) Give the NFA that recognizes $L((30 \cup 75 \cup 45)^* \circ 10)$ as it would be constructed according to the proof of Lemma 1.29 from the lecture.

(b) Give another NFA with at most 5 states that recognizes the same language (you do not have to justify your answer).