Exercise 6.1 (DFA, 1+1+2 marks)
Consider the following two DFAs (deterministic finite automata) with \( \Sigma = \{0, 1\} \):

(a) What languages \( L_1 \) and \( L_2 \) do these two automata individually recognize?

(b) Give the formal definition for \( M_1 \).

(c) Show that \( L_1 \cup L_2 \) is also a regular language, by constructing one DFA. Please hand in a high quality diagram.

Exercise 6.2 (DFA, 1+1 marks)

(a) Construct a DFA that recognizes the language \( L \) with an alphabet \( \Sigma = \{0, 1\} \), where \( L = \{w \mid w \) has both an even number of 0’s and an even number of 1’s\}.

(b) Give the state diagram for a DFA accepting the language \( L = \{w \mid w \) starts with 1 and contains 10 or starts with 0 and contains the 01\}.
   The alphabet is \( \Sigma = \{0, 1\} \).
Exercise 6.3 (Regular Languages, 2.5 + 1.5 marks)

In this exercise we want to prove that regular languages are closed under intersection and under complement. The intersection of two languages is defined as \( L_1 \cap L_2 \). The complement of a language is defined as the set of all words in \( \Sigma^* \) which are not in \( L \), i.e. \( \overline{L} = \Sigma^* \setminus L \) (\( \Sigma^* \) is the set of all words/strings over \( \Sigma \)).

Let \( L \) and \( L' \) be regular languages that are recognized by DFAs \( M = (Q, \Sigma, \delta, q_0, F) \) and \( M' = (Q', \Sigma', \delta', q'_0, F') \), respectively.

(a) Show that the regular languages are closed under intersection, i.e. give a finite automaton that recognizes \( L \cap L' \).

(b) Show that the regular languages are closed under complement, i.e. give a finite automaton that recognizes \( \overline{L} \).