Exercise 2.1 (Propositional Logic, 2.5 + 2.5 marks)

\[ \phi = (A \leftrightarrow \neg B) \land \neg (C \lor B \rightarrow A) \]
\[ I = \{ A \mapsto \bot, B \mapsto \top, C \mapsto \bot \} \]

(a) Show that \( \phi \equiv \neg A \land B \) by using the equivalences from the lectures and the equivalences \( \psi \land \neg \psi \equiv \bot \) and \( \psi \lor \bot \equiv \psi \). Apply in each step only one of the equivalences with the exception that you may implicitly use associativity.

(b) Consider a vocabulary with only four atomic propositions \( A, B, C, D \). How many models are there for the following formulae? Explain.

i) \( (A \land B) \lor (B \land C) \)

ii) \( (A \leftrightarrow B) \land (B \leftrightarrow C) \)

Exercise 2.2 (Propositional Logic, 5 marks)

Determine the validity or invalidity of the following argument:

“If Alice is elected class-president, then either Betty is elected vice-president, or Carol is elected treasurer. Betty is elected vice-president. Therefore if Alice is elected class-president, then Carol is not elected treasurer.”

Please explain every formal step.