Introduction to Multi-Agent Programming

12. Voting

Preferences, Voting Protocols, Borda Protocol, Arrow’s Impossibility Result

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**Voting**

**Introduction**

- In open systems agents have their *individual preferences*
- Agreements can be reached by *voting*
  - Applicable for both *benevolent* and *self-interested* agents
- A *voting system* derives a social preference form each individual preference
- How to find a fair solution? What means a *fair solution*?
- One way to approach the *fairness problem* is to require:
  - If one agent prefers A to B and another one prefers B to A then their votes should cancel each other out
  - If one agent’s preferences are A,B,C and another one’s are B,C,A and a third one prefers C,A,B then their votes should cancel out
Voting
Definition

- Given a set of agents $A$ and a set of outcomes $O$, each agent $I \in A$ has a strict, asymmetric, and transitive preference relation $\succ_i$ on $O$
- A voting system derives a social preference $\succ^*$ form all agents' individual preferences ($\succ_i,...,\succ_{|A|}$)
- Desired properties of a voting system are:
  1. $\succ^*$ exists for all possible inputs $\succ_i$
  2. $\succ^*$ should be defined for every pair $o, o' \in O$
  3. $\succ^*$ should be asymmetric and transitive over $O$
  4. The outcome should be Pareto efficient: if $\forall i \in A, o \succ_i o'$ then $o \succ^* o'$, e.g., if all agents prefer beer over milk then $\succ^*$ should also prefer beer over milk
  5. The scheme should be independent of irrelevant alternatives, i.e. when adding another alternative the ranking should be same
  6. No dictatorship: if $o \succ_i o'$ implies $o \succ^* o'$ for all preferences of the other agents
Voting
Example

15 mathematicians are planning to throw a party. They must first decide which beverage the department will serve at this party. There are three choices available to them: beer, wine, and milk.

- 6 x Milk > Wine > Beer
- 5 x Beer > Wine > Milk
- 4 x Wine > Beer > Milk
Voting
Plurality protocol

• Majority voting protocol where alternatives are compared simultaneously

• In the example:
  – Each one votes for her/his favorite drink
  – The drink with the most votes is the winner
  – Beer would get 5 votes, wine 4, and milk 6 → Milk wins!

  Problems:
  • There are 8 agents that prefer beer over milk and wine over milk, but only 6 that have the opposite preferences, and yet milk wins?
  • Irrelevant alternatives can lead to different results
Voting
Binary Voting

• Alternatives are voted on pairwise, the winner stays to challenge further alternatives while the looser is eliminated

• For example:
  – beer & wine: wine wins, wine & milk: wine wins

• Problems:
  – Irrelevant alternatives can lead to different results
  – The order of the considered pairings can totally change the outcome. For example:

35% of agents have preferences  a > d > b > a
33% of agents have preferences  a > c > d > b
32% of agents have preferences  b > a > c > d
**Voting**

**Borda Protocol**

- Takes into account all agents’ knowledge equally
- Let $|O|$ denote the number of alternatives
- Assigns $|O|$ points to an alternative whenever it is highest in some agent’s preference, assigns $|O-1|$ whenever it is second, ...
- Counts are summed across voters, alternative with highest count becomes the social choice

- In the example:
  - Milk: $6 \times 3 + 5 \times 1 + 4 \times 1 = 27$
  - Wine: $6 \times 2 + 5 \times 2 + 4 \times 3 = 34$
  - Beer: $6 \times 1 + 5 \times 3 + 4 \times 2 = 29$
  - Wine wins!
Voting
Arrow’s impossibility Theorem

• There is no voting mechanism that satisfies all six conditions \(\text{(Arrow, 1951)}\)
  – For example, also in the Borda protocol, irrelevant alternatives can lead to paradox results (violating (5)):

<table>
<thead>
<tr>
<th>Agent</th>
<th>Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a \succ b \succ c \succ d)</td>
</tr>
<tr>
<td>2</td>
<td>(b \succ c \succ d \succ a)</td>
</tr>
<tr>
<td>3</td>
<td>(c \succ d \succ a \succ b)</td>
</tr>
<tr>
<td>4</td>
<td>(a \succ b \succ c \succ d)</td>
</tr>
<tr>
<td>5</td>
<td>(b \succ c \succ d \succ a)</td>
</tr>
<tr>
<td>6</td>
<td>(c \succ d \succ a \succ b)</td>
</tr>
<tr>
<td>7</td>
<td>(a \succ b \succ c \succ d)</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th></th>
<th>Borda count</th>
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<tbody>
<tr>
<td></td>
<td>c wins with 20, b has 19, a has 18, d loses with 13</td>
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<tr>
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<th>Borda count with (d) removed</th>
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<tbody>
<tr>
<td></td>
<td>a wins with 15, b has 14, c loses with 13</td>
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Winner turns loser and loser turns winner paradox in the Borda protocol
Summary

• Voting methods have to be implemented carefully with respect to the desired outcome
• In practice, the plurality protocol is often used in multi-agent systems
• However, the Borda protocol should be preferred as it can effectively aggregate multiple disparate opinions