Introduction to Multi-Agent Programming

12. Voting

Preferences, Voting Protocols, Borda Protocol, Arrow's Impossibility Result

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Voting

Introduction

- In open systems agents have their *individual preferences*
- Agreements can be reached by *voting*
  - Applicable for both *benevolent* and *self-interested* agents
- A voting system derives a social preference form each individual preference
- How to find a fair solution? What means a fair solution?
- One way to approach the fairness problem is to require:
  - If one agent prefers A to B and another one prefers B to A then their votes should cancel each other out
  - If one agent’s preferences are A,B,C and another one’s are B,C,A and a third one prefers C,A,B then their votes should cancel out

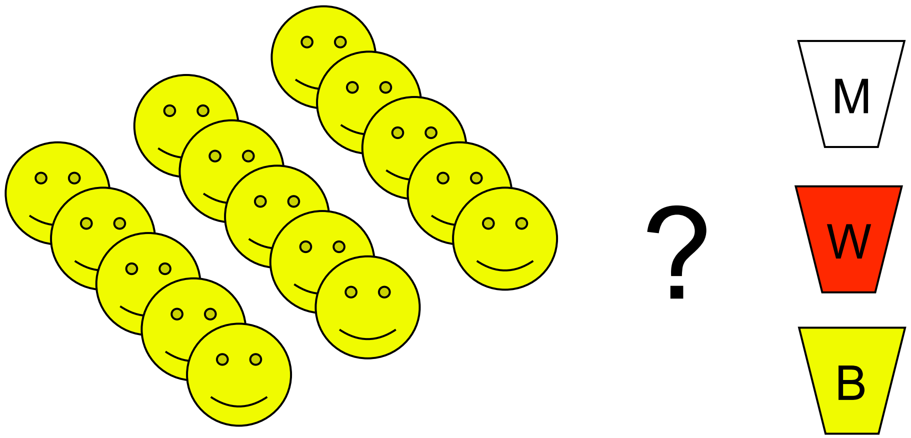
Voting

Definition

- Given a set of agents A and a set of outcomes O, each agent $I \in A$ has a strict, asymmetric, and transitive preference relation $\succ_i$ on O
- A voting system derives a social preference $\succ_*$ form all agents` individual preferences ($\succ_i$,…, $\succ_{|A|}$)
- Desired properties of a voting system are:
  1. $\succ_*$ exists for all possible inputs $\succ_i$
  2. $\succ_*$ should be defined for every pair o, $o' \in O$
  3. $\succ_*$ should be asymmetric and transitive over O
  4. The outcome should be Pareto efficient: if $\forall i \in A$, $o \succ_i o'$ then $o \succ_* o'$, e.g., if all agents prefer beer over milk then $\succ_*$ should also prefer beer over milk
  5. The scheme should be independent of irrelevant alternatives, i.e. when adding another alternative the ranking should be same
  6. No dictatorship: if $o \succ_i o'$ implies $o \succ_* o'$ for all preferences of the other agents

Voting
Example

15 mathematicians are planning to throw a party. They must first decide which beverage the department will serve at this party. There are three choices available to them: beer, wine, and milk.



6 x Milk $\succ$ Wine $\succ$ Beer

5 x Beer $\succ$ Wine $\succ$ Milk

4 x Wine $\succ$ Beer $\succ$ Milk

Voting

Plurality protocol

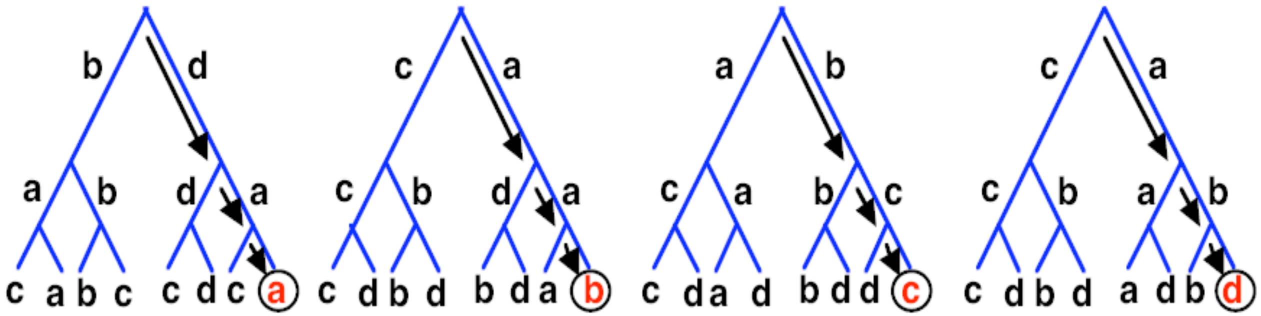
- Majority voting protocol where alternatives are compared simultaneously

- In the example:
  - Each one votes for her/his favorite drink
  - The drink with the most votes is the winner
  - Beer would get 5 votes, wine 4, and milk 6 → Milk wins!
  - Problems:
    - There are 8 agents that prefer beer over milk and wine over milk, but only 6 that have the opposite preferences, and yet milk wins?
    - Irrelevant alternatives can lead to different results

Voting
Binary Voting

- Alternatives are voted on pairwise, the winner stays to challenge further alternatives while the looser is eliminated
- For example:
  - beer & wine: wine wins, wine & milk: wine wins
- Problems:
  - Irrelevant alternatives can lead to different results
  - The order of the considered pairings can totally change the outcome. For example:

35% of agents have preferences $a \succ d \succ b \succ a$
33% of agents have preferences $a \succ c \succ d \succ b$
32% of agents have preferences $b \succ a \succ c \succ d$



Voting
Borda Protocol

- Takes into account all agents' knowledge equally
- Let |O| denote the number of alternatives
- Assigns |O| points to an alternative whenever it is highest in some agent's preference, assigns |O-1| whenever it is second, …
- Counts are summed across voters, alternative with highest count becomes the social choice

- In the example:
  - Milk: 6*3 + 5*1 + 4*1 = 27
  - Wine: 6*2 + 5*2 + 4*3 = 34
  - Beer: 6*1 + 5*3 + 4*2 = 29
  - Wine wins!

Voting

Arrow's impossibility Theorem

- There is no voting mechanism that satisfies all six conditions (Arrow, 1951)
  - For example, also in the Borda protocol, irrelevant alternatives can lead to paradox results (violating (5)):

Agent	Preferences
1	$a \succ b \succ c \succ d$
2	$b \succ c \succ d \succ a$
3	$c \succ d \succ a \succ b$
4	$a \succ b \succ c \succ d$
5	$b \succ c \succ d \succ a$
6	$c \succ d \succ a \succ b$
7	$a \succ b \succ c \succ d$
Borda count	c wins with 20, b has 19, a has 18, d loses with 13
Borda count with d removed	a wins with 15, b has 14, c loses with 13

Winner turns loser and loser turns winner paradox in the Borda protocol

Summary

- Voting methods have to be implemented carefully with respect to the desired outcome
- In practice, the plurality protocol is often used in multi-agent systems
- However, the Borda protocol should be preferred as it can effectively aggregate multiple disparate opinions