Exercise 14.1 (P, 1.5 + 0.5 marks)

(a) Show that $P$ is closed under union, intersection, and complement.

(b) The complexity class $\text{coP}$ contains all languages $L$ whose complement is in $P$. Formally, $\text{coP} = \{L \mid \overline{L} \in P\}$. Is $P = \text{coP}$?

Exercise 14.2 (NP, 2 + 2 marks)

(a) The language $\text{DOUBLESAT}$ is defined as

$$\text{DOUBLESAT} = \{\langle \phi \rangle \mid \phi \text{ is a propositional formula that has at least two satisfying assignments}\}.$$ 

Show that $\text{DOUBLESAT} \in \text{NP}$.

(b) The language $\text{SETPACK}$ is defined as

$$\text{SETPACK} = \{\langle C, k \rangle \mid C \text{ is a finite collection of finite sets, with at least } k \text{ sets being disjoint}\}.$$ 

Show that $\text{SETPACK} \in \text{NP}$.

Exercise 14.3 (Reduction, 2 + 2 marks)

A clique in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge. An independent set in an undirected graph is a subset of its vertices such that for any two vertices in the subset there is no edge connecting them. A vertex cover of an undirected graph is a set of vertices such that each edge of the graph is incident to at least one vertex of the set. Consider the following three languages:

$$\text{CLIQUE} = \{\langle G, k \rangle \mid G \text{ is an undirected graph that contains a clique with } k \text{ vertices}\}$$

$$\text{INDSET} = \{\langle G, k \rangle \mid G \text{ is an undirected graph that contains an independent set with } k \text{ vertices}\}$$

$$\text{VERTEXCOVER} = \{\langle G, k \rangle \mid G \text{ is an undirected graph that has a vertex cover with } k \text{ vertices}\}$$

(a) Show that $\text{CLIQUE}$ is polynomially reducible to $\text{INDSET}$.

(b) Show that $\text{INDSET}$ is polynomially reducible to $\text{VERTEXCOVER}$.