Exercise Sheet 12
Due: January 27, 2010

Exercise 12.1 (Decidable Languages, 0.5 + 0.5 + 0.5 + 0.5 marks)
Consider the DFA \( M = (Q, \Sigma, \delta, q_0, F) \) with \( Q = \{q_0, q_1, q_2\} \), \( \Sigma = \{0, 1\} \), \( F = \{q_0\} \) and \( \delta \) being given by the table:

\[
\begin{array}{c|cc}
\delta & 0 & 1 \\
q_0 & q_0 & q_1 \\
q_1 & q_2 & q_2 \\
q_2 & q_0 & q_1 \\
\end{array}
\]

(a) Is \( \langle M, 0100 \rangle \in A_{DFA} \)?
(b) Is \( \langle M, 011 \rangle \in A_{DFA} \)?
(c) Is \( \langle M \rangle \in E_{DFA} \)?
(d) Is \( \langle M, M \rangle \in EQ_{DFA} \)?

Justify your answers!

Exercise 12.2 (Decidable Languages, 1.5 + 1.5 + 1.5 marks)
Show that the following languages are decidable:
(a) \( EQ_{DFA, RE} = \{ \langle D, R \rangle \mid D \text{ is a DFA and } R \text{ is a regular expression and } L(D) = L(R) \} \)
(b) \( A_{CFG} = \{ \langle G \rangle \mid G \text{ is a CFG that generates } \epsilon \} \)
(c) \( ALL_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA that recognizes } \Sigma^* \} \)

Exercise 12.3 (Undecidable Languages, 4 marks)
Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

\[ N = \{ \langle M, w \rangle \mid M \text{ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on } w \}. \]

Show that \( N \) is undecidable. *Hint: Use a reduction from \( A_{TM} \).*