Exercise 6.1 (DFA, 1+1+2 marks)
Consider the following two DFAs (deterministic finite automata) with $\Sigma = \{0, 1\}$:

(a) What languages ($L_1$ and $L_2$) do these two automata individually recognize?

(b) Give the formal definition for $M_1$.

(c) Show that $L_1 \cup L_2$ is also a regular language, by constructing one DFA. Please hand in a high quality diagram.

Hint: Use the definition of the union automaton from the proof that the regular languages are closed under the union operation and remove all states (and transitions) that are not reachable from the start state.

Exercise 6.2 (DFA and NFA, 1+1 marks)

(a) Construct a DFA that recognizes the language $L = \{w \mid \text{the length of } w \text{ is at most 3}\}$ with an alphabet $\Sigma = \{0, 1\}$.

(b) Give the state diagram for an NFA (non-deterministic finite automaton) with at most four states accepting the language $L = \{w \mid w \text{ ends with } 01 \text{ or with } 10\}$. The alphabet is $\Sigma = \{0, 1\}$.

Exercise 6.3 (Regular Languages, 2.5 + 1.5 marks)

In this exercise we want to prove that regular languages are closed under intersection and under complement. The intersection of two languages is defined as $L_1 \cap L_2$. The complement of a language is defined as the set of all words in $\Sigma^*$ which are not in $L$, i.e. $\bar{L} = \Sigma^* \setminus L$ ($\Sigma^*$ is the set of all words/strings over $\Sigma$).

Let $L$ and $L'$ be regular languages that are recognized by DFAs $M = (Q, \Sigma, \delta, q_0, F)$ and $M' = (Q', \Sigma', \delta', q'_0, F')$, respectively.

(a) Show that the regular languages are closed under intersection, i.e. give a finite automaton that recognizes $L \cap L'$.

(b) Show that the regular languages are closed under complement, i.e. give a finite automaton that recognizes $L$. 