

Introduction to Multi-Agent Programming

7. Cooperative Sensing

Modeling Sensors, Kalman Filter,
Markov Localization, Potential Fields

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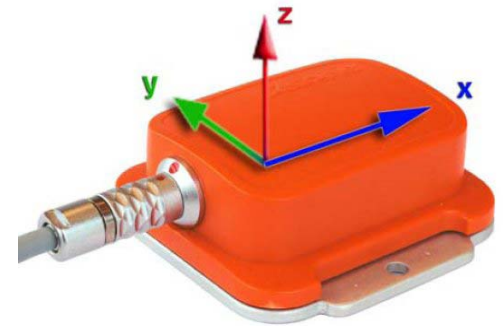
Introduction

- Cooperative sensing and world modeling follows the goal of **deliberative decision making**
 - Which is in **contrast** to reactive acting
- Agents perceive their environment by **sensors**
 - However, sensing can either be inaccurate or ambiguous
 - Sensing requires the process of world modeling for meaningful and robust **decision making**
 - **Probabilistic models** are the first choice for this task
- World models can be used to extract abstract **predicates** of the world
 - For example, "***objectInOpponentGoal(ball)***"

Processing Sensor data I

Inertial Measurement Unit (IMU)

- A **closed system** for detecting orientation and motion of a vehicle or human
- Typically consists of 3 accelerometers, 3 gyroscopes, and 3 magnetometers
- **Data rate** @100 Hz
- Gyro data reliable only within some time period (**temperature drift**)
- Magnetometer data can locally be wrong (**magnetic fields**)
- Therefore, gyro, accelerometer, and magnetometer data is **fused** by a Kalman Filter onboard the IMU sensor
- For the estimation of robot poses (x, y, θ) also **wheel odometry**, i.e. a hardware counting the number of wheel revolutions per second, is required



Processing Sensor data II

Laser Range Finders (LRFs)

- Found on nearly every robot
- **Highly accurate**, high data rate
- Measures **distances and angles** to surrounding objects
- Returns distances d_i and angles α_i , with $i \in [0 \dots \text{FOV}/\text{resolution}]$

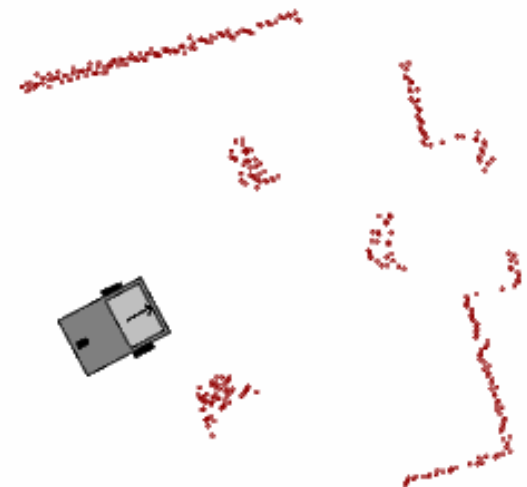
	Sick LMS200	Sick S300	Hokuyo URG-04LX
Weight	4500 g	1200 g	160 g
Volume	$\approx 20 \text{ cm}^3$	$\approx 15 \text{ cm}^3$	$\approx 5 \text{ cm}^3$
FOV	180°	270°	240°
Max. Range	80 m	30 m	4 m
Max. Ang. Res.	0.25°	0.5°	0.36°
Accuracy	$\pm 15 \text{ mm}$	$\pm 30 \text{ mm}$	$\pm 10 \text{ mm}$
Scans per second	30	20	10
Interface	RS-232/RS-422	RS-232/RS-422	RS-232/USB



SICK LMS200



Hokuyo URG



Scan taken on a soccer field

Processing Sensor data III

Color Cameras

- Sensor that generates **color images**, e.g. with 640x480 pixel resolution @30hz
- Can be used for **ball detection**
 - **Color thresholding**, e.g. separation of ball colors from background
 - Determination of relative object location by camera calibration or **interpolation**



Logitech Quickcam
4000 Pro



Sony DFW-500

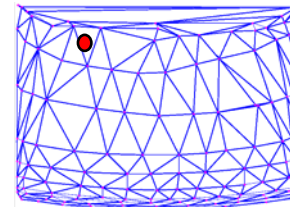
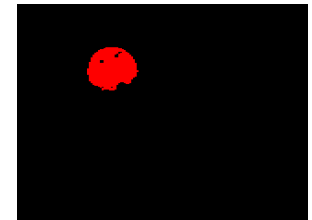
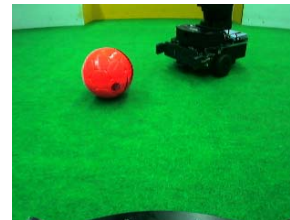
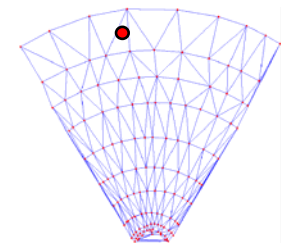


Image coordinates



World coordinates

Modeling Sensor noise

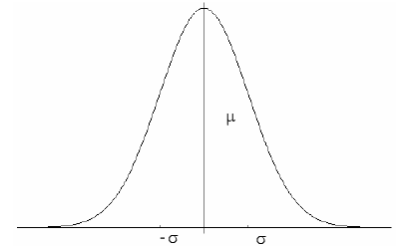
- Sensor data is typically **noisy**, .e.g., the distance measurement of a LRF at one meter can be $1\text{m} \pm 1\text{cm}$
- Sensor noise is typically modeled by a **normal distribution**
- Model fully described by **mean** μ and **variance** σ^2

one-dimensional:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2}$$

Notation:

$$x \sim N(\mu_x, \sigma_x^2)$$

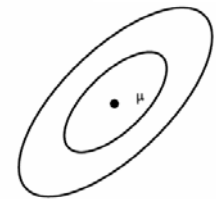


N-dimensional:

$$p(x) = \frac{1}{\sqrt{(2\pi)^n \det(\Sigma_x)}} e^{-\frac{1}{2}(x-\mu_x)^T \Sigma_x^{-1} (x-\mu_x)}$$

Notation:

$$x \sim N(\mu_x, \Sigma_x)$$



Transformation of density functions I

Linear Transformation

- When processing data from multiple sensors, all observations have to be transformed into a **single coordinate system**
- For example, distance and angle measurements of a LRF have to be integrated into a **Cartesian** coordinate frame
- Linear transformations can be represented by $F(u) = Au + b$, where A is a $n \times m$ Matrix and b a **n-dimensional** vector
- **Mean** μ_x and **covariance** Σ_x can then be computed by:

$$\begin{aligned}\mu_x &= E(x) & \Sigma_x &= E((x - E(x))(x - E(x))^T) \\ &= E(Au + b) & &= E((Au + b - AE(u) - b)(Au + b - AE(u) - b)^T) \\ &= AE(u) + b & &= E((A(u - E(u)))(A(u - E(u)))^T) \\ &= A\mu_u + b & &= E((A(u - E(u))((u - E(u))^T A^T)) \\ & & &= AE((u - E(u))(u - E(u))^T)A^T \\ & & &= A\Sigma_u A^T\end{aligned}$$

Transformation of density functions II

Non-Linear Transformation

- Linearization necessary in order to yield **normal distribution**
 - Approximation by **Taylor polynom** while skipping higher order terms:

$$F(u) \approx F(\hat{u}) + \nabla F(\hat{u})(u - \hat{u})$$

where $\nabla F(\hat{u}) = \frac{\partial F}{\partial u}(\hat{u})$ is a $n \times m$ Matrix (also known as **Jacobi- Matrix**) with partial derivatives of F at \hat{u} .

- Notation according to **linear case**:

$$A = \nabla F(\mu_u)$$

$$b = F(\mu_u) - \nabla F(\mu_u)\mu_u$$

- Mean** μ_x and **covariance** Σ_x can then be computed by:

$$\mu_x = A\mu_u + b$$

$$= \nabla F(\mu_u)\mu_u + F(\mu_u) - \nabla F(\mu_u)\mu_u$$

$$= F(\mu_u)$$

$$\Sigma_x = A\Sigma_u A^T$$

$$= \nabla F(\mu_u)\Sigma_u \nabla F(\mu_u)^T$$

Transformation of density functions III

Example: LRF Measurement Transformation

- We assume a **normally distributed** error of distance measurement d and angle measurement α :

$$d \sim N(\mu_d, \sigma_d^2) \quad \alpha \sim N(\mu_\alpha, \sigma_\alpha^2)$$

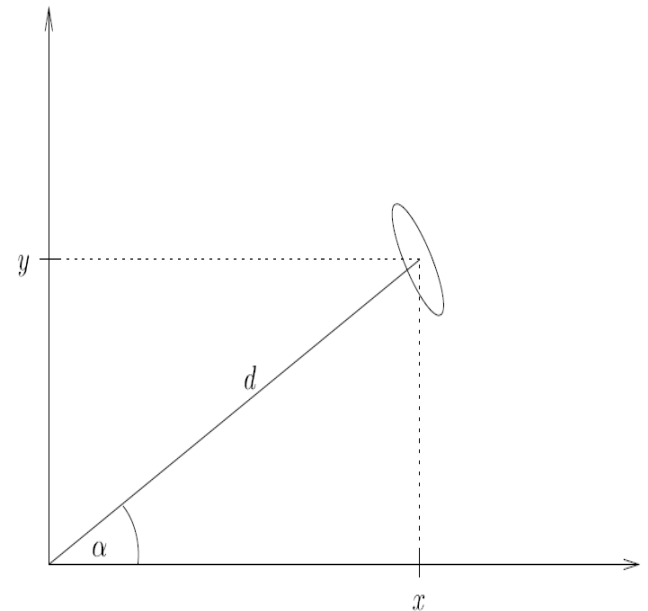
- Transformation** function F :

$$F\left(\begin{pmatrix} d \\ \alpha \end{pmatrix}\right) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d \cos \alpha \\ d \sin \alpha \end{pmatrix}$$

$$\Sigma_{xy} = \nabla F_{d\alpha} \Sigma_{d\alpha} \nabla F_{d\alpha}^T$$

$$\nabla F_{d\alpha} = \begin{pmatrix} \cos \alpha & -d \sin \alpha \\ \sin \alpha & d \cos \alpha \end{pmatrix}$$

$$\Sigma_{d\alpha} = \begin{pmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_\alpha^2 \end{pmatrix}$$



Transformation of density functions IV

Example: LRF Measurement Transformation

- Assume $d=3000\text{mm}$, $\alpha=30^\circ$, $\sigma_d=100\text{mm}$, $\sigma_\alpha=5.7^\circ$

$$\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} = \begin{pmatrix} d \cos \alpha \\ d \sin \alpha \end{pmatrix} = \begin{pmatrix} 2598 \\ 1500 \end{pmatrix}$$

$$\nabla F_{d\alpha} = \begin{pmatrix} \cos \alpha & -d \sin \alpha \\ \sin \alpha & d \cos \alpha \end{pmatrix} = \begin{pmatrix} 0.866 & -1500 \\ 0.500 & 2598 \end{pmatrix}$$

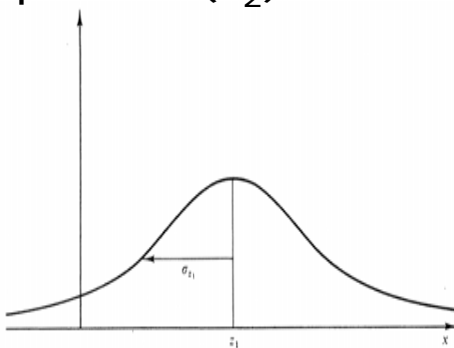
$$\Sigma_{d\alpha} = \begin{pmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_\alpha^2 \end{pmatrix} = \begin{pmatrix} 10000 & 0 \\ 0 & 0.01 \end{pmatrix}$$

$$\Sigma_{xy} = \nabla F_{d\alpha} \Sigma_{d\alpha} \nabla F_{d\alpha}^T = \begin{pmatrix} 30000 & -34640 \\ -34640 & 70000 \end{pmatrix}$$

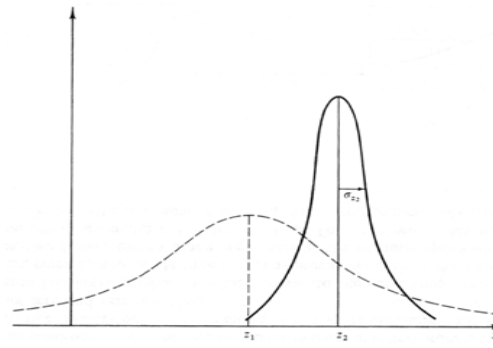
Kalman Filter I

Introduction

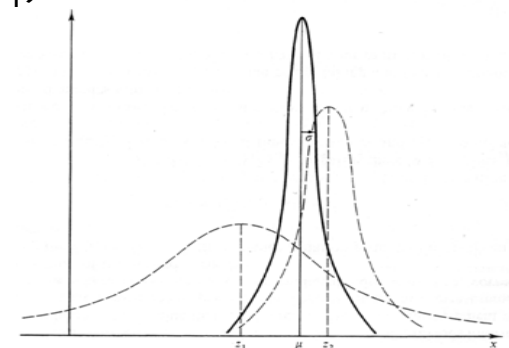
- An *optimal recursive data processing algorithm*
 - *optimal* since it processes all data regardless of precision
 - *recursive* since the filter does not need to keep all data in memory and to reprocess it
- Fusion of two **independent** measurements of the **same concept**
- Each measurement has a **confidence** expressed by the variance of the Gaussian
- Example: two people on a boat estimate their 1D location. The 2nd person (z_2) is more skilled than the 1st one (z_1)



Conditional density
of observation z_1



Conditional density
of observation z_2



Conditional density after
combining z_1 and z_2

Kalman Filter II

Update Formula

one-dimensional:

$$l_1 \sim N(\mu_1, \sigma_1^2)$$

$$l_2 \sim N(\mu_2, \sigma_2^2)$$

$$l = \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} \left(\frac{1}{\sigma_1^2} l_1 + \frac{1}{\sigma_2^2} l_2 \right)$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

n-dimensional:

$$m_1 \sim N(\mu_1, \Sigma_1)$$

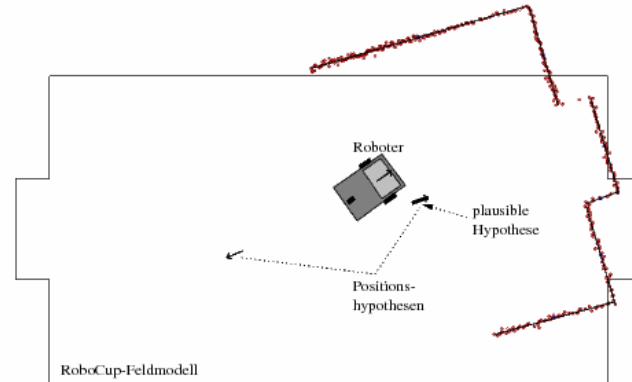
$$m_2 \sim N(\mu_2, \Sigma_2)$$

$$m = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1} (\Sigma_1^{-1} m_1 + \Sigma_2^{-1} m_2)$$

$$\Sigma = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$$

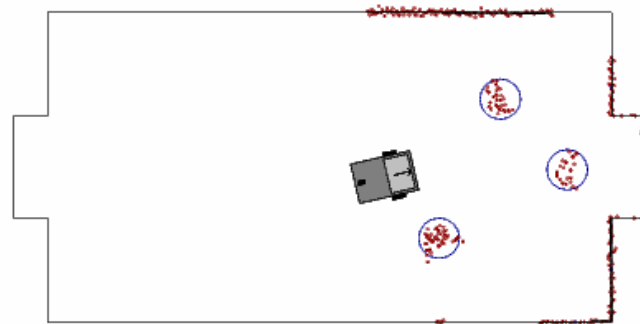
Case-Study: Cooperative opponent sensing on a soccer field I

1st step: players estimate their own position and orientation on the field by matching scans with the field model



Matching a scan to field model

2nd step: Extraction of other players by discarding scan points belonging to field walls and clustering the remaining ones. For each cluster the center of gravity is assumed to correspond to the center of another robot.



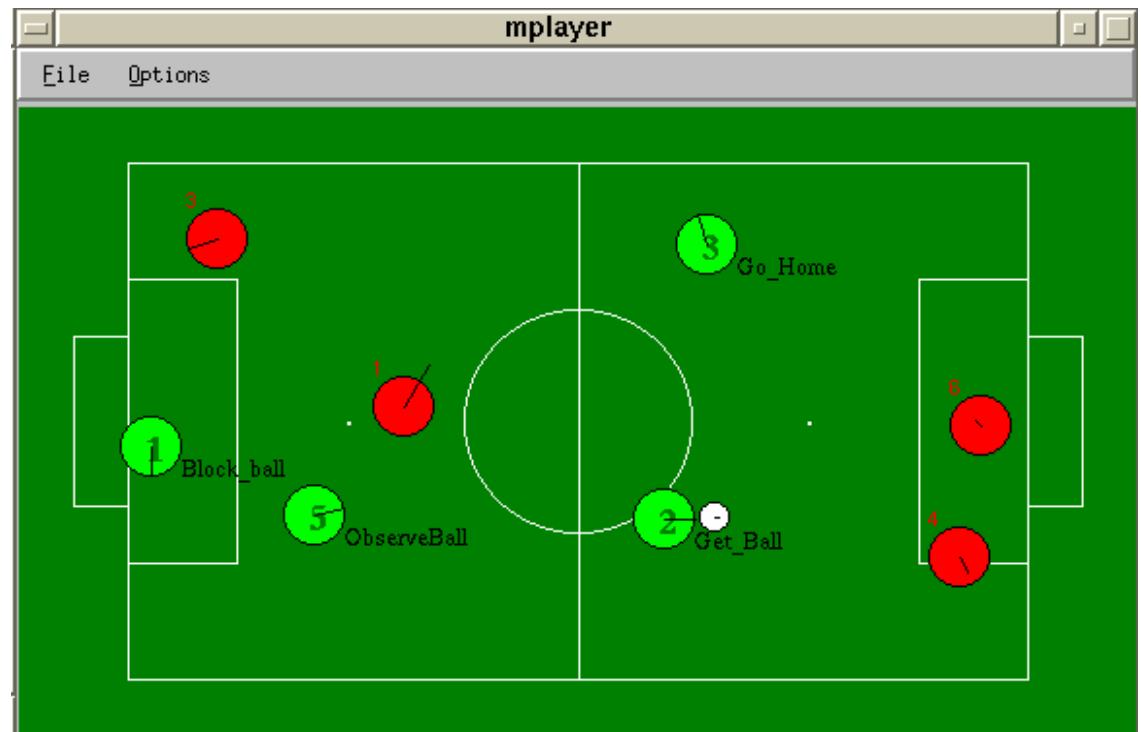
Extracting and clustering objects

Case-Study: Cooperative opponent sensing on a soccer field II

3rd step:

Communication of position, heading and velocity of each detected object and own pose to a central multi-sensor integration module

4th step: Assignment of team player IDs to objects → Detection of opponents (red)



Cooperative world model

Integration of multiple measurements I

State representation:

Each observation is modeled by a **random variable**: $\mathbf{x}_s = (x_s, y_s, \theta_s, v_s, \omega_s)^T$

With mean $\hat{\mathbf{x}}_s$ and covariance Σ_s , where (x_s, y_s) is the position, θ_s the orientation, and v_s, ω_s are the **translational and rotational velocities*** of the object.

Modeling of the **covariance**: $\Sigma_s = \text{diag}(\sigma_{x_s}^2, \sigma_{y_s}^2, \sigma_{\theta_s}^2, \sigma_{v_s}^2, \sigma_{\omega_s}^2)$,

where $\sigma_{x_s}, \sigma_{y_s}, \sigma_{\theta_s}, \sigma_{v_s}, \sigma_{\omega_s}$ are **constant standard deviations** determined experimentally


State projection:

$$\hat{\mathbf{x}}_r \leftarrow F_s(\hat{\mathbf{x}}_r, t) = \begin{pmatrix} \hat{x}_r + \cos(\hat{\theta}_r) \hat{v}_r t \\ \hat{y}_r + \sin(\hat{\theta}_r) \hat{v}_r t \\ \hat{\theta}_r + \hat{\omega}_r t \\ \hat{v}_r \\ \hat{\omega}_r \end{pmatrix}$$

$$\Sigma_r \leftarrow \nabla F_s \Sigma_r \nabla F_s^T + \Sigma_a(t)$$

$$\Sigma_a(t) = \text{diag}(\sigma_{x_a}^2 t, \sigma_{y_a}^2 t, \sigma_{\theta_a}^2 t, \sigma_{v_a}^2 t, \sigma_{\omega_a}^2 t)$$

Constant standard
deviations
determined
experimentally



*Note velocities are determined by differencing the last 10 pose estimates

Integration of multiple measurements II

State update:

(a) Observation of
a **new object**:

$$\hat{\mathbf{x}}_r = \hat{\mathbf{x}}_s, \quad \Sigma_r = \Sigma_s$$

(b) Observation of
a **known object**:

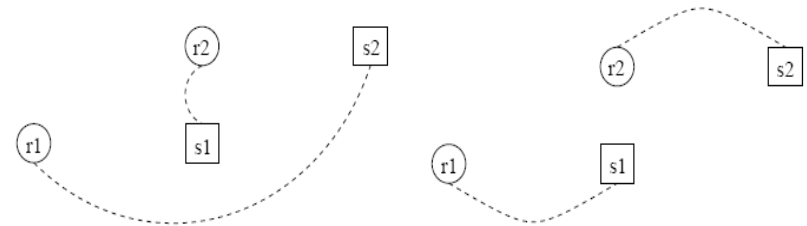
$$\begin{aligned}\hat{\mathbf{x}}_r &\leftarrow (\Sigma_r^{-1} + \Sigma_s^{-1})^{-1} (\Sigma_r^{-1} \hat{\mathbf{x}}_r + \Sigma_s^{-1} \hat{\mathbf{x}}_s) \\ \Sigma_r &\leftarrow (\Sigma_r^{-1} + \Sigma_s^{-1})^{-1}\end{aligned}$$

Data association problem, i.e. how to associate observations to known objects?

Greedy method:

Search the global world model for the track whose predicted mean is **closest to the observation**. Assign observation if distance is beyond a certain threshold.

→ Can be **sub-optimal**!



Greedy method

Optimal solution

Better approach: *geometric assignment*

Go over all possible sets of assignment pairs (s_i, r_i)

Find assignment that **minimizes**
$$\sum_{i=1}^n \text{dist}(s_i, r_i)^2$$

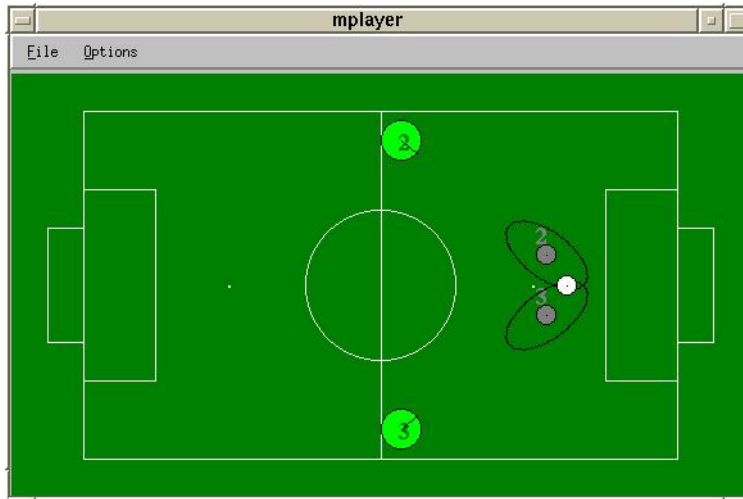
Single Object Tracking from Noisy Data

Example: Ball Tracking

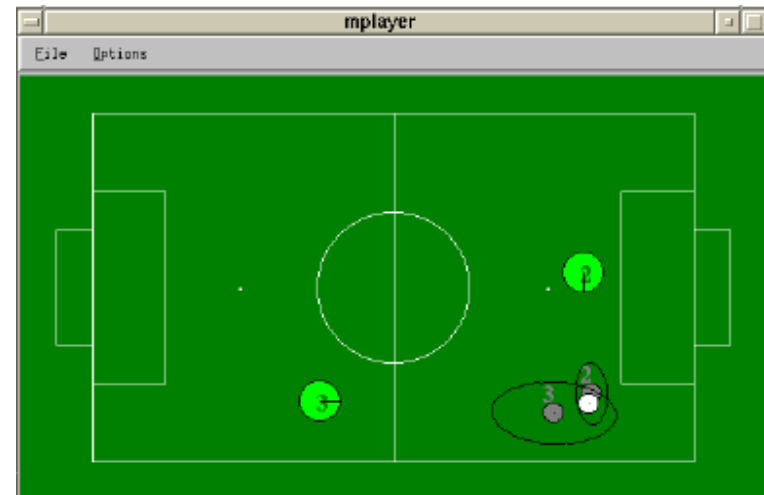
- For example, **global ball position estimation**: stereo vision with robot groups
- Detection of the ball by **vision**, e.g. detecting the ball by color
 - Estimation of the **angle** is quite accurate, however, **distance** is not
 - Kalman Filter **integration** yields an error ellipse with respect to these confidences
 - Fusion of two estimates respects error ellipse: effect of **"triangulation"**
- **Prediction step** (predict next location where ball will be observed):
 - Project ball position into the future using a constant negative ball acceleration
 - Consider a certain projection error
- **Update step** (when new observation is made):
 - Integrate new measurement (using a weighted average on the error)
 - distance error grows with distance
 - angular error is small and constant

Single Object Tracking from Noisy Data

Example: Ball Tracking

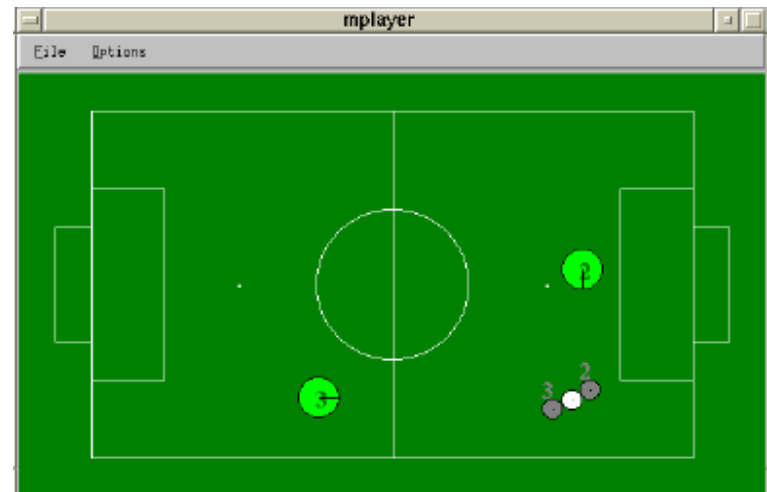


Effect of triangulation



Kalman filtering

Kalman filtering
compared to simple
averaging: highly
confident estimates are
more strongly weighted



Simple averaging

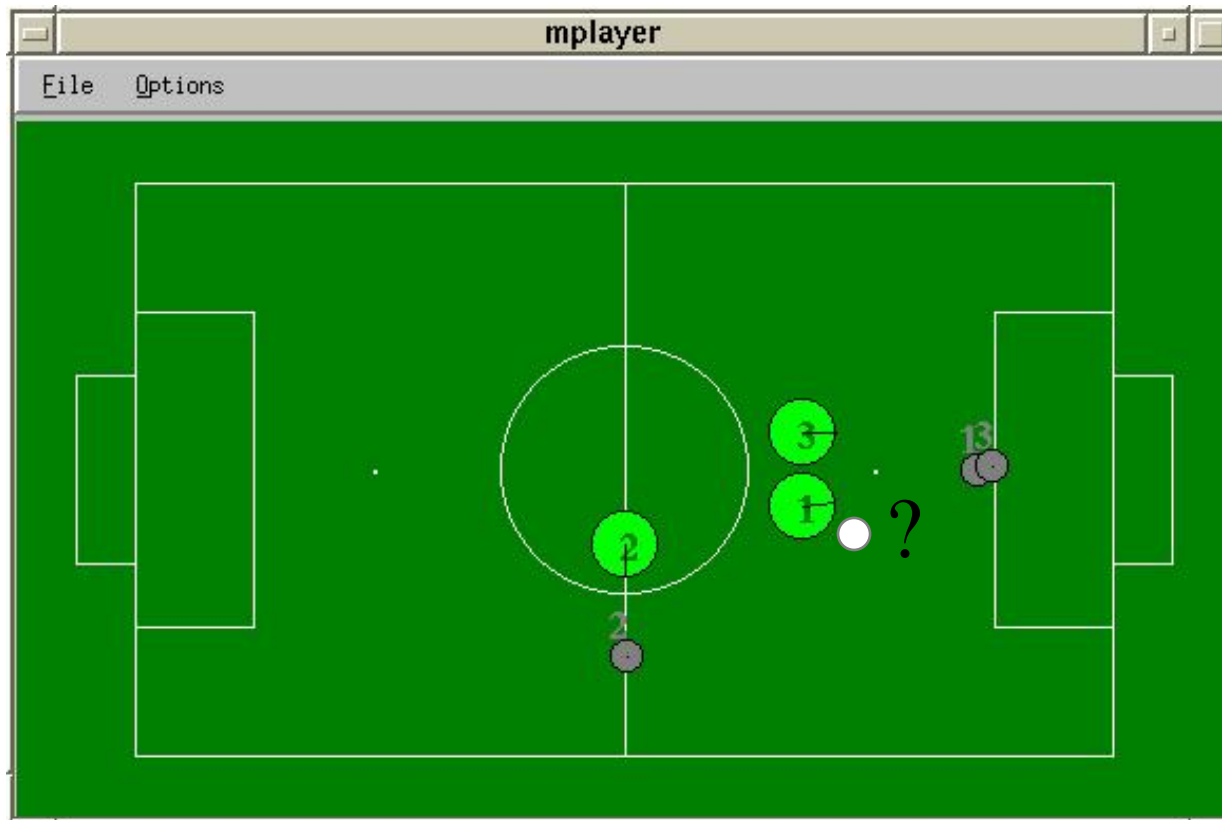
The Importance of Global Ball Estimation



Minho (Portugal) shoots at our goal from the other side of the field. Our goalie gets this information **early on from his team mates** and can easily defend

Single Object Tracking from Noisy Data

Problem of false positives (ghost balls)



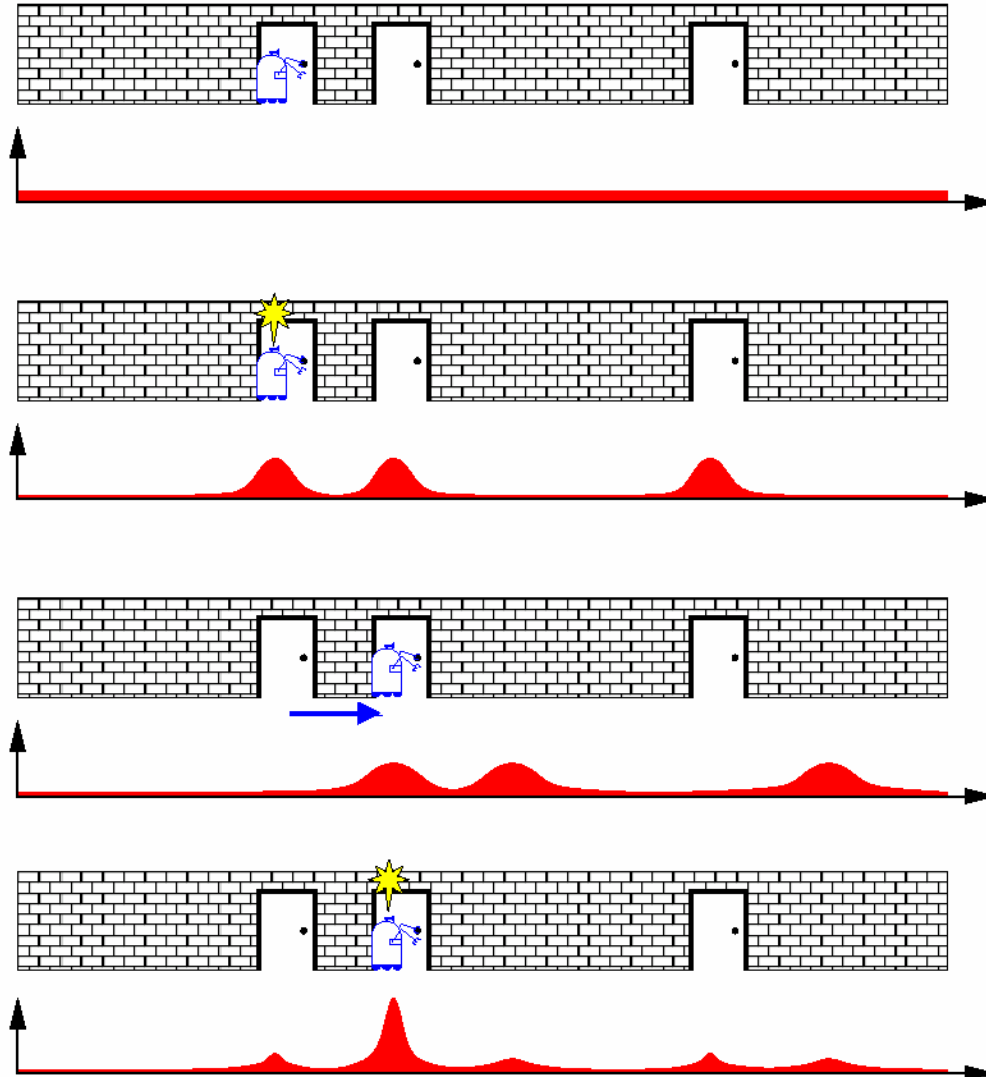
Player 2 is hallucinating

Markov Localization as Observation Filter

Introduction

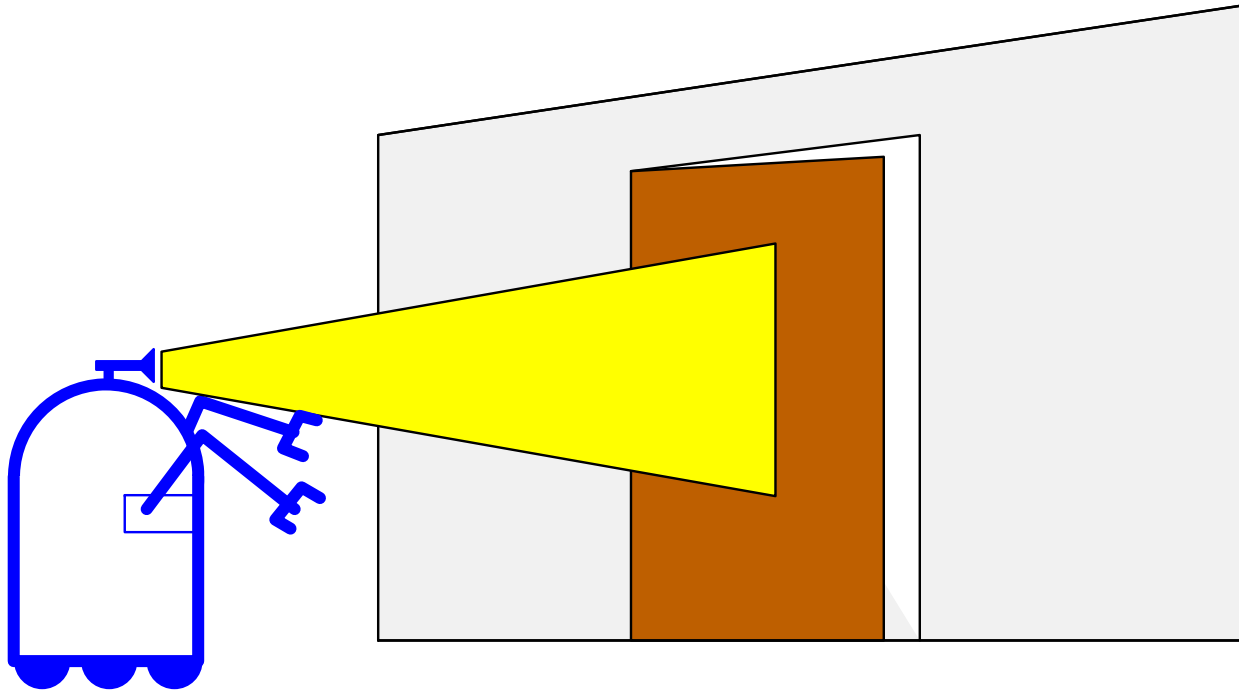
- The Kalman-Filter expects that measurements originate from the **same objects**
 - However, color thresholding on a soccer field might confuse “**red t-shirts**” with the ball
 - Consequently, Kalman filtering yields **poor results**
- Markov localization: Simultaneous tracking of **multiple hypotheses**
- Idea: To **filter-out** false positives with probability grid

Probabilistic Localization



Simple Example of State Estimation

- Suppose a robot obtains measurement z
- What is $P(open/z)$?



Causal vs. Diagnostic Reasoning

- $P(open/z)$ is diagnostic.
- $P(z/open)$ is causal.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge:

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

Example

- $P(z/open) = 0.6$ $P(z/\neg open) = 0.3$
- $P(open) = P(\neg open) = 0.5$

$$P(open | z) = \frac{P(z | open)P(open)}{P(z | open)p(open) + P(z | \neg open)p(\neg open)}$$

$$P(open | z) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

z raises the probability that the door is open.

Markov Localization as Observation Filter

Prediction & Update I

- **Discretization** of the soccer field into a two-dimensional (x,y) grid

- Each cells of the grid reflects the probab $p(z) = \frac{1}{\text{\# cells}}$ in the cell

- **Uniform initialization** of the grid before any observation is processed

- **Prediction step:**

- Simple model of ball motion $p(z) \leftarrow \sum_{z'} p(z | z') p(z')$ where $p(z | z')$ denotes the probability that the ball is at z given it was at z' .

- When assuming that all kind of motion directions are equally possible, and velocities are **normally distributed** with zero mean and covariance σ_v^2 , $p(z | z')$ can be modeled by a time-depended Gaussian around z' : $p(z | z') \sim N(z', \text{diag}(\sigma_v^2 t, \sigma_v^2 t))$

Markov Localization as Observation Filter

Prediction & Update II

- Update step:

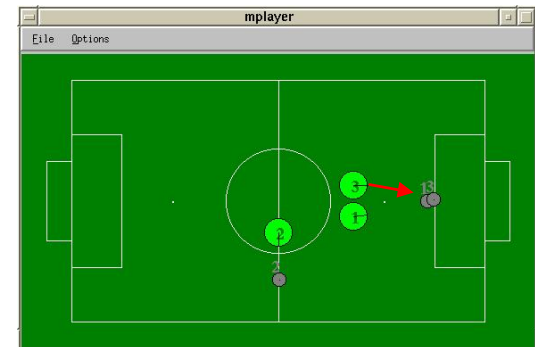
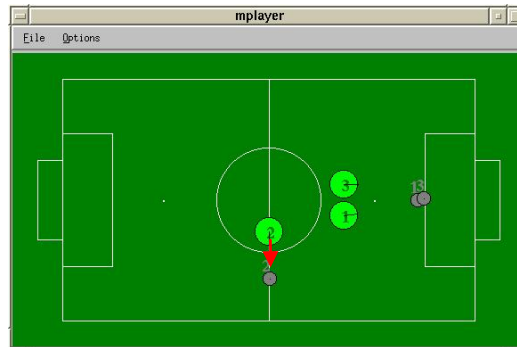
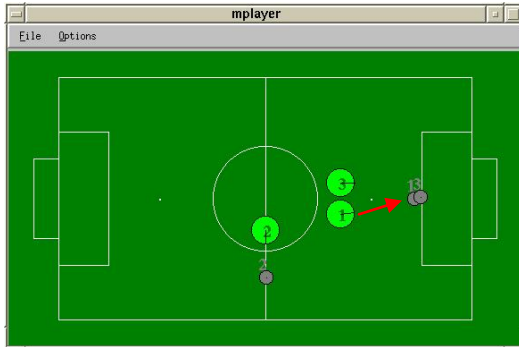
- Fusion of new ball observation z_b into the grid according to Bayes' law:

$$p(z) \leftarrow \frac{p(z_b | z)p(z)}{\sum_{z'} p(z_b | z')p(z')}$$

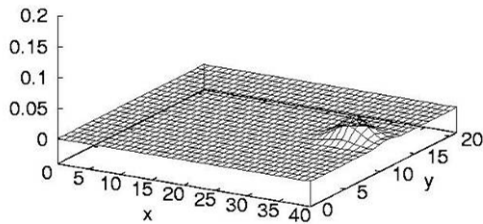
Normalization: Ensuring that probabilities sum up to 1.0

- The sensor model $p(z_b | z)$ determines the likelihood observing z_b given the ball is at position z .
 - e.g. less confidence as more far away the ball
- Finally, the Markov grid can be used for outlier rejection
 - Kalman filtering is only applied at the highest peak of the distribution
 - If another peak becomes more likely, the Kalman filter is re-initialized accordingly

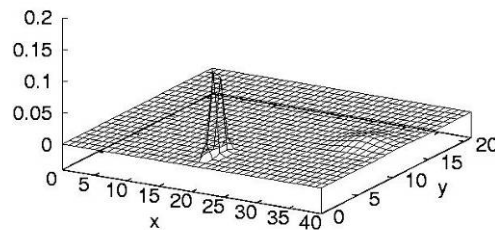
Phantom Balls: Development of Probability Distribution I



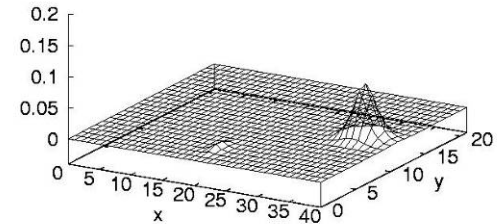
$p(x,y)$ after 1st measurement (1)



$p(x,y)$ after 2nd measurement (2)

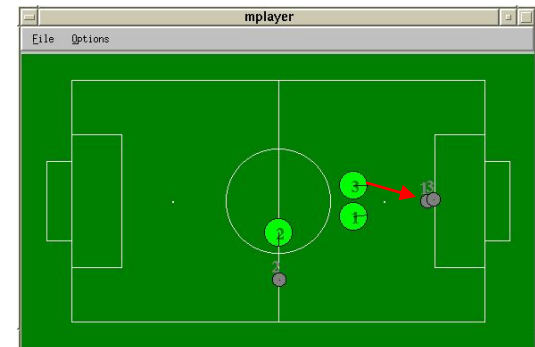
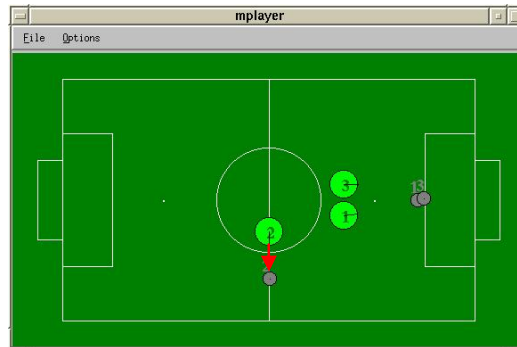
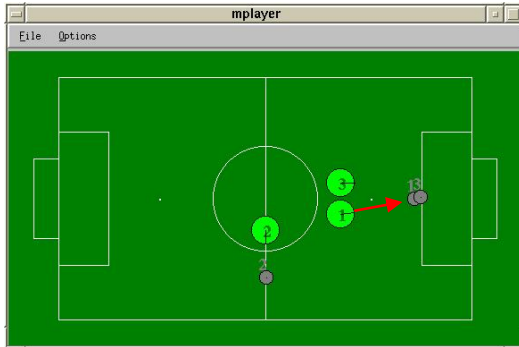


$p(x,y)$ after 3rd measurement (3)

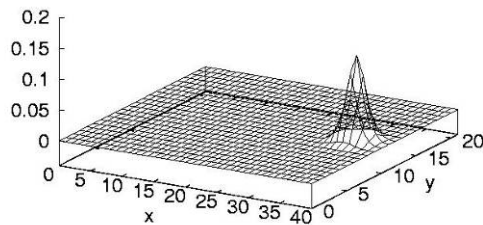


Consider area with highest peak as possible ball area
and use KF there

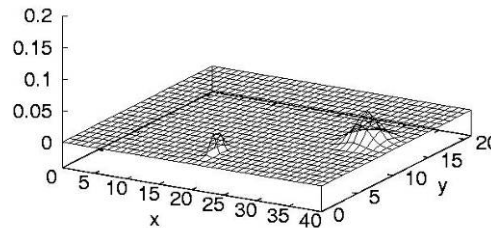
Phantom Balls: Development of Probability Distribution II



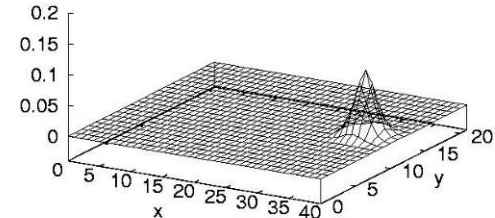
$p(x,y)$ after 4th measurement (1)



$p(x,y)$ after 5th measurement (2)



$p(x,y)$ after 6th measurement (3)



At *RoboCup 2000*, 938 out of 118388 (0.8%) ball observations were ignored because of the Markov localization filter.

Demo Webplayer

See www.cs-freiburg.de

Potential Fields

Introduction

- Originally introduced for robot path planning
 - Robot is considered as particle within a force field, the potential field
 - Potential field is generated by overlaying repulsive potentials (e.g. obstacles) and attractive potentials (e.g. goals)
 - The motion of the robot is determined by negating the field's gradient, leading to the potential minimum
 - Repulsive and attractive potentials are computed separately
- Can also be used for strategic decision making (e.g. CS-Freiburg)

Potential Fields

Potentials

- Potentials are **differentiable** functions of the type $U : \mathcal{C}_{free} \rightarrow \mathbf{R}$, where \mathcal{C}_{free} is the set of robot configurations
- Typically, high values indicate obstacles and low values goals
- Given differentiable potentials, one can compute the force at each **configuration** \mathbf{q} by:

$$\vec{F}(\mathbf{q}) = -\vec{\nabla}U(\mathbf{q})$$

- For example, given a 2D work space, force $F(\mathbf{q})$ can be computed from $U(\mathbf{q})$ by:

$$\vec{F}(\mathbf{q}) = - \left(\begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{array} \right) U(\mathbf{q}) = - \left(\begin{array}{c} \frac{\partial U(\mathbf{q})}{\partial x} \\ \frac{\partial U(\mathbf{q})}{\partial y} \end{array} \right)$$

Potential Fields

Attractive Potential

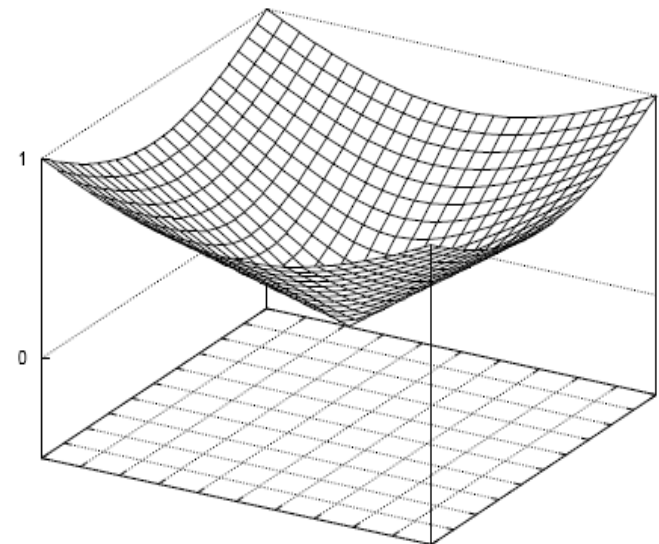
- Influence of potential has to be workspace wide!
- **Linearly decreasing** potential with increasing distance to goal \mathbf{q}_{ziel} :

$$U_{\text{att}}(\mathbf{q}) = \xi \rho(\mathbf{q}), \text{ with } \rho(\mathbf{q}) = \|\mathbf{q} - \mathbf{q}_{\text{ziel}}\| \text{ and scaling factor } \xi.$$

Computation of **force** F_{att} :

$$\begin{aligned}\vec{F}_{\text{att}}(\mathbf{q}) &= -\xi \vec{\nabla} \rho(\mathbf{q}) \\ &= -\xi (\mathbf{q} - \mathbf{q}_{\text{ziel}}) / \|\mathbf{q} - \mathbf{q}_{\text{ziel}}\|\end{aligned}$$

- **Singularity** at $\mathbf{q} = \mathbf{q}_{\text{ziel}}$!
 - Has to be dealt with separately



Attractive potential

Potential Fields

Repulsive Potential

- Influence of potential can be **limited** in order to simplify computations
- Increasing potential with increasing **distance to object**:

$$U_{rep}(\mathbf{q}) = \begin{cases} \frac{1}{2}\eta \left(\frac{1}{\rho(\mathbf{q})} - \frac{1}{\rho_0} \right)^2 & \text{falls } \rho(\mathbf{q}) \leq \rho_0 \\ 0 & \text{falls } \rho(\mathbf{q}) > \rho_0 \end{cases}$$

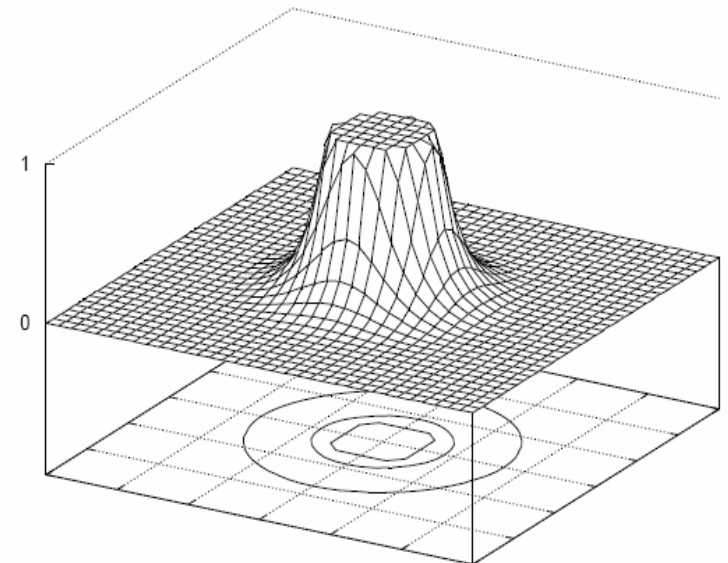
Where η is a scaling factor, $p(\mathbf{q})$ the distance to the obstacle, and p_0 the maximal influence radius of the potential

The distance function should respect the **shape** of the object, for example:

$$\rho(\mathbf{q}) = \min_{\mathbf{q}' \in CB} \|\mathbf{q} - \mathbf{q}'\|$$

Computation of **force** F_{rep} :

$$\vec{F}_{rep}(\mathbf{q}) = \begin{cases} \eta \left(\frac{1}{\rho(\mathbf{q})} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(\mathbf{q})} \vec{\nabla} \rho(\mathbf{q}) & \text{falls } \rho(\mathbf{q}) \leq \rho_0 \\ 0 & \text{falls } \rho(\mathbf{q}) > \rho_0 \end{cases}$$



Repulsive potential

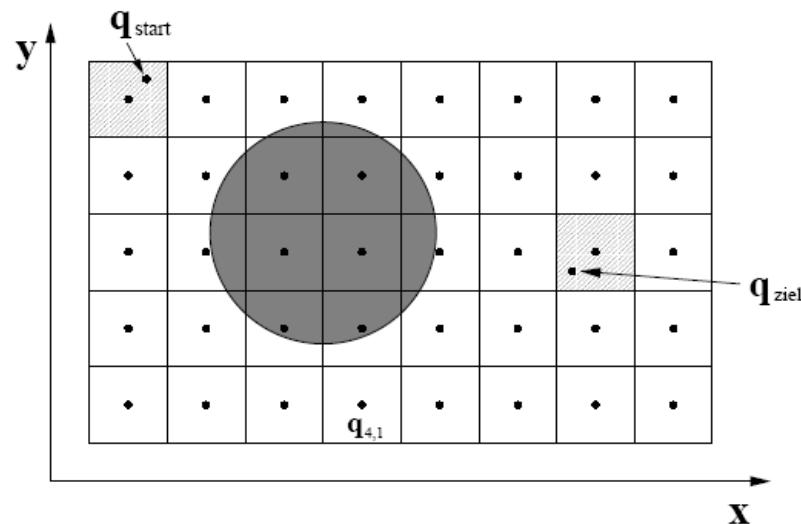
Potential Fields

Grid representation

- **Discretization** of the configuration space into equally sized cells
- **Grid representation** GC is defined for every $\mathbf{q}=(x,y)$ as follows:

$$GC = \{\mathbf{q} \in \mathcal{C} \mid \mathbf{q} = (i\delta_x, j\delta_y), i = 1, \dots, N, j = 1, \dots, M\}$$

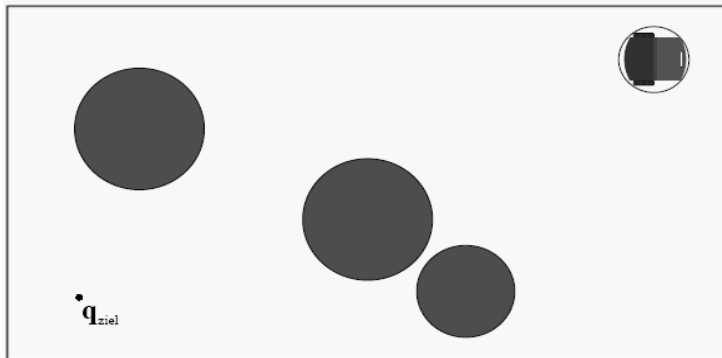
where δ_x, δ_y are the **step sizes** in X and Y direction, and N, M are the number of cells along the axes, respectively



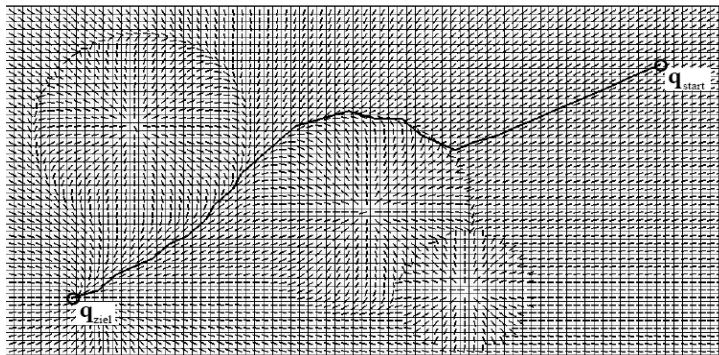
Potential Fields

Computing the potential field

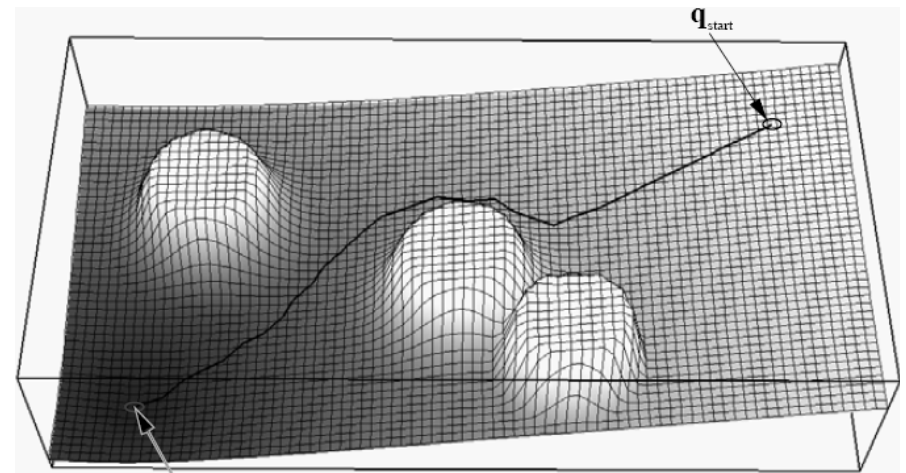
Computation by:
$$U(\mathbf{q}) = \sum_{i=1}^M U_{att_i}(\mathbf{q}_{goal_i}) + \sum_{j=1}^N U_{rep_j}(\mathbf{q})$$



Situation: Obstacles, start, and goal



Resulting force field



Resulting potential field

Case-Study: Extracting predicates for playing soccer I

- Predicates are the basis for **action selection** and **strategic decision** making
- Can be considered as **world model abstractions**
- Simple **predicates** of objects (can be directly computed from positions):
 - ***InOpponentsGoal(object)***
 - Object in opponent goal?
 - ***InOwnGoal(object)***
 - Object in own goal?
 - ***CloseToBorder(object)***
 - The distance to any border is beyond a threshold?
 - ***FrontClear()***
 - Neither another object nor the border is in front?
 - ***InDefense(object)***
 - Object in the last third of the soccer field?

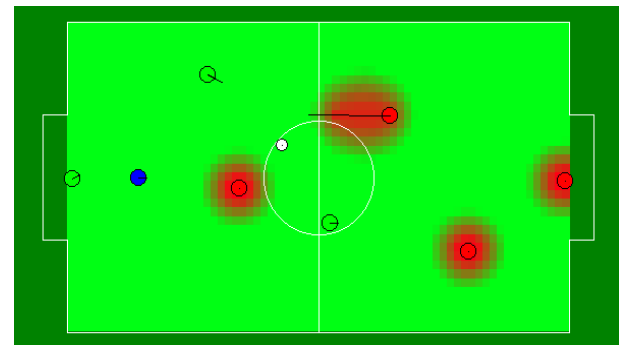
Case-Study: Extracting predicates for playing soccer II

- Extended predicates:

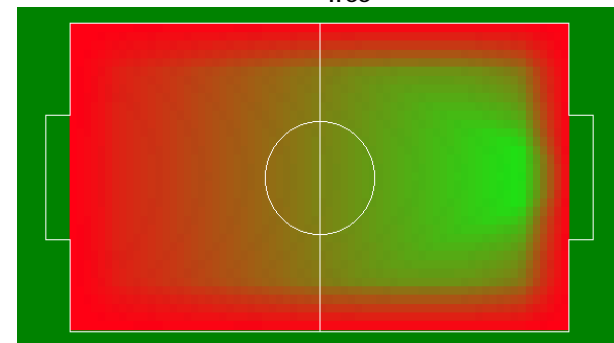
- computed by normalized potential fields:
($f_i: \mathbb{R} \times \mathbb{R} \rightarrow [0..1]$)
- discretized by grids of 10x10cm cell size

- Examples:

- f_{free} : indicates positions under the influence of the opponent
- f_{covered} : indicates position covered by teammates
- f_{desired} : indicates tactical good positions



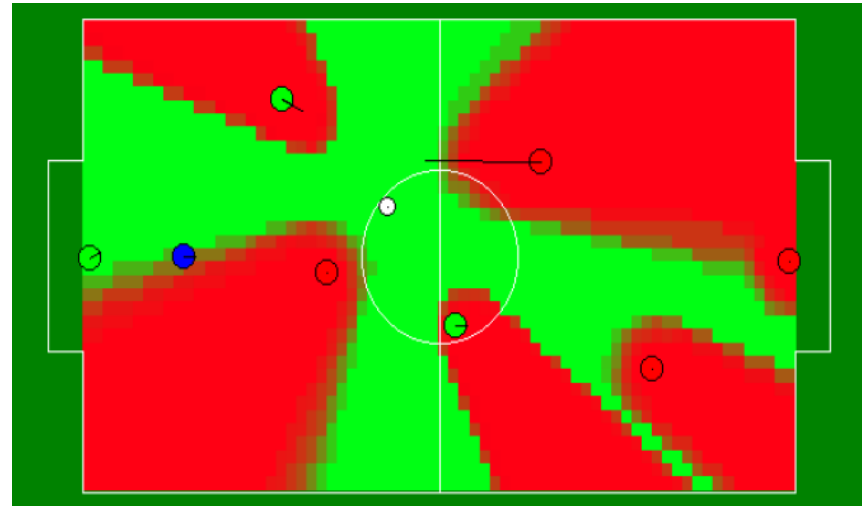
f_{free}



f_{desired}

Case-Study: Extracting predicates for playing soccer III

- Combined potential fields:
 - $f_{ballview}$: indicates whether the direct line from the ball to a position is free
 - Recursive computation:



$$f_{ballview}(k(z_1)) = 1$$

$$f_{ballview}(k(z_i)) = f_{ballview}(k(z_{i-1})) \cdot f_{free}(k(z_i)) \cdot (1 - f_{covered}(k(z_i)))$$

Where z_1, \dots, z_n are the indices of the cells on the line

Summary

- Consistent world models are the key to **deliberative** acting!
- The Kalman Filter is a tool for **accurately** estimating object poses
 - However, only **single hypotheses** can be tracked
- Markov Localization is a tool for **robust** object tracking by considering multiple hypotheses
 - However, **accuracy depends** on the discretization
- Best results are yielded by **combining** both methods
- Potential Fields are an efficient tool for generating **predicates** from complex representations, simplifying decision making of a mobile agent

Literature

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 - Peter Maybeck ***Stochastic Models, Estimation, and Control***, Volume 1, Academic Press, Inc.
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- Markov Localization:
 - S. Thrun, W. Burgard, and D. Fox ***Probabilistic Robotics***. MIT-Press, 2005.
- Cooperative Sensing:
 - M. Dietl, J.-S. Gutmann and B. Nebel, **Cooperative Sensing in Dynamic Environments**, in: *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'01)*, Maui, Hawaii, 2001
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