Principles of AI Planning
6. State-space search: search algorithms

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Our plan for the next lectures

Choices to make:

1. search direction: progression/regression/both
   \(\rightarrow\) previous chapter

2. search space representation: states/sets of states
   \(\rightarrow\) previous chapter

3. search algorithm: uninformed/heuristic; systematic/local
   \(\rightarrow\) this chapter

4. search control: heuristics, pruning techniques
   \(\rightarrow\) next chapters
Search algorithms are used to find solutions (plans) for transition systems in general, not just for planning tasks.

Planning is one application of search among many.

In this chapter, we describe some popular and/or representative search algorithms, and (the basics of) how they apply to planning.

Most of this is review of material that should be known (details: Russell and Norvig’s textbook).
Search states vs. search nodes

In search, one distinguishes:

- **search states** $s \rightsquigarrow$ states (vertices) of the transition system
- **search nodes** $\sigma \rightsquigarrow$ search states plus information on where/when/how they are encountered during search

**What is in a search node?**

Different search algorithms store different information in a search node $\sigma$, but typical information includes:

- $\text{state}(\sigma)$: associated search state
- $\text{parent}(\sigma)$: pointer to search node from which $\sigma$ is reached
- $\text{action}(\sigma)$: an action/operator leading from $\text{state}(\text{parent}(\sigma))$ to $\text{state}(\sigma)$
- $g(\sigma)$: cost of $\sigma$ (length of path from the root node)

For the root node, $\text{parent}(\sigma)$ and $\text{action}(\sigma)$ are undefined.
Search states $\neq$ (planning) states:

- **Search states** don’t have to correspond to states in the planning sense.
  - progression: search states $\approx$ (planning) states
  - regression: search states $\approx$ sets of states (formulae)

- Search algorithms for planning where search states are planning states are called state-space search algorithms.

- Strictly speaking, regression is not an example of state-space search, although the term is often used loosely.

- However, we will put the emphasis on progression, which is almost always state-space search.
A general search algorithm can be applied to any transition system for which we can define the following three operations:

- **init()**: generate the *initial state*
- **is-goal(s)**: test if a given state is a *goal state*
- **succ(s)**: generate the set of *successor states* of state $s$, along with the *operators* through which they are reached (represented as pairs $\langle o, s' \rangle$ of operators and states)

Together, these three functions form a *search space* (a very similar notion to a transition system).
Search for planning: progression

Let $\Pi = \langle A, I, O, G \rangle$ be a planning task.

**Search space for progression search**

**states:** all states of $\Pi$ (assignments to $A$)
- $\text{init}() = I$
- $\text{succ}(s) = \{\langle o, s' \rangle \mid o \in O, s' = \text{app}_o(s)\}$
- $\text{is-goal}(s) = \begin{cases} \text{true} & \text{if } s \models G \\ \text{false} & \text{otherwise} \end{cases}$
Let \( \langle A, I, O, G \rangle \) be a planning task.

**Search space for regression search**

**states:** all formulae over \( A \)

- \( \text{init}() = G \)
- \( \text{succ}(\phi) = \{ \langle o, \phi' \rangle | o \in O, \phi' = \text{regr}_o(\phi), \phi' \) is satisfiable} \)
  (modified if splitting is used)
- \( \text{is-goal}(\phi) = \begin{cases} \text{true} & \text{if } I \models \phi \\ \text{false} & \text{otherwise} \end{cases} \)
uninformed search vs. heuristic search:

- **uninformed search algorithms** only use the basic ingredients for general search algorithms
- **heuristic search algorithms** additionally use **heuristic functions** which estimate how close a node is to the goal

systematic search vs. local search:

- **systematic algorithms** consider a large number of search nodes simultaneously
- **local search algorithms** work with one (or a few) candidate solutions (search nodes) at a time
- not a black-and-white distinction; there are **crossbreeds** (e.g., enforced hill-climbing)
uninformed vs. heuristic search:

- For **satisficing** planning, heuristic search vastly outperforms uninformed algorithms on most domains.
- For **optimal** planning, the difference is less pronounced. An efficiently implemented uninformed algorithm is not easy to beat in most domains.

systematic search vs. local search:

- For **satisficing** planning, the most successful algorithms are somewhere between the two extremes.
- For **optimal** planning, systematic algorithms are required.
Before we describe the different search algorithms, we introduce three procedures used by all of them:

- **make-root-node**: Create a search node without parent.
- **make-node**: Create a search node for a state generated as the successor of another state.
- **extract-solution**: Extract a solution from a search node representing a goal state.
Procedure make-root-node

**make-root-node**: Create a search node without parent.

```python
def make-root-node(s):
    σ := new node
    state(σ) := s
    parent(σ) := undefined
    action(σ) := undefined
    g(σ) := 0
    return σ
```
make-node: Create a search node for a state generated as the successor of another state.

```python
def make-node(σ, o, s):
    σ' := new node
    state(σ') := s
    parent(σ') := σ
    action(σ') := o
    g(σ') := g(σ) + 1
return σ'
```
**Procedure extract-solution**

**extract-solution**: Extract a solution from a search node representing a goal state.

```python
# Procedure extract-solution

def extract-solution(σ):
    solution := new list
    while parent(σ) is defined:
        solution.push-front(action(σ))
        σ := parent(σ)
    return solution
```
Uninformed search algorithms

- Uninformed algorithms are less relevant for planning than heuristic ones, so we keep their discussion brief.
- Uninformed algorithms are mostly interesting to us because we can compare and contrast them to related heuristic search algorithms.

Popular uninformed systematic search algorithms:
- breadth-first search
- depth-first search
- iterated depth-first search

Popular uninformed local search algorithms:
- random walk
Breadth-first search without duplicate detection

Breadth-first search

\[
\text{queue} := \textbf{new} \ \text{fifo-queue} \\
\text{queue}.\text{push-back}(\text{make-root-node}(\text{init}())) \\
\textbf{while not} \ \text{queue}.\text{empty}(): \\
\quad \sigma = \text{queue}.\text{pop-front}() \\
\quad \textbf{if} \ \text{is-goal}(\text{state}(\sigma)):\ \\
\quad \quad \textbf{return} \ \text{extract-solution}(\sigma) \\
\quad \textbf{for each} \ \langle o, s \rangle \in \textbf{succ}(\text{state}(\sigma)):\ \\
\quad \quad \sigma' := \text{make-node}(\sigma, o, s) \\
\quad \quad \text{queue}.\text{push-back}(\sigma') \\
\textbf{return} \ \text{unsolvable}
\]

- Possible improvement: duplicate detection (see next slide).
- Another possible improvement: test if \( \sigma' \) is a goal node; if so, terminate immediately. (We don’t do this because it obscures the similarity to some of the later algorithms.)
Breadth-first search with duplicate detection

```plaintext
queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed := ∅
while not queue.empty():
    σ = queue.pop-front()
    if state(σ) ∉ closed:
        closed := closed ∪ {state(σ)}
        if is-goal(state(σ)):
            return extract-solution(σ)
        for each ⟨o, s⟩ ∈ succ(state(σ)):
            σ' := make-node(σ, o, s)
            queue.push-back(σ')
return unsolvable
```
Breadth-first search with duplicate detection

```plaintext
queue := new fifo-queue
queue.push-back(make-root-node(init()))
closed := ∅
while not queue.empty():
    σ = queue.pop-front()
    if state(σ) ∉ closed:
        closed := closed ∪ {state(σ)}
        if is-goal(state(σ)):
            return extract-solution(σ)
    for each ⟨o, s⟩ ∈ succ(state(σ)):
        σ' := make-node(σ, o, s)
        queue.push-back(σ')
return unsolvable
```
Random walk

\[ \sigma := \text{make-root-node}(\text{init}()) \]

forever:

if \( \text{is-goal}(\text{state}(\sigma)) \):

return \( \text{extract-solution}(\sigma) \)

Choose a random element \( \langle o, s \rangle \) from \( \text{succ}(\text{state}(\sigma)) \).

\[ \sigma := \text{make-node}(\sigma, o, s) \]

- The algorithm usually does not find any solutions, unless almost every sequence of actions is a plan.
- Often, it runs indefinitely without making progress.
- It can also fail by reaching a \textbf{dead end}, a state with no successors. This is a weakness of many local search approaches.
Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular systematic heuristic search algorithms:
- greedy best-first search
- A*
- weighted A*
- IDA*
- depth-first branch-and-bound search
- breadth-first heuristic search
- ...
Heuristic search algorithms are the most common and overall most successful algorithms for classical planning.

Popular heuristic local search algorithms:
- hill-climbing
- enforced hill-climbing
- beam search
- tabu search
- genetic algorithms
- simulated annealing
- ...
Heuristic search: idea

distance estimate

init

goal

Heuristic search: idea
A heuristic search algorithm requires one more operation in addition to the definition of a search space.

**Definition (heuristic function)**

Let $\Sigma$ be the set of nodes of a given search space. A **heuristic function** or **heuristic** (for that search space) is a function $h : \Sigma \rightarrow \mathbb{N}_0 \cup \{\infty\}$.

The value $h(\sigma)$ is called the **heuristic estimate** or **heuristic value** of heuristic $h$ for node $\sigma$. It is supposed to estimate the distance from $\sigma$ to the nearest goal node.
What exactly is a heuristic estimate?

What does it mean that \( h \) “estimates the goal distance”?

- For most heuristic search algorithms, \( h \) does not need to have any strong properties for the algorithm to work (= be correct and complete).

- However, the **efficiency** of the algorithm closely relates to how accurately \( h \) reflects the actual goal distance.

- For some algorithms, like A*, we can prove strong formal relationships between properties of \( h \) and properties of the algorithm (optimality, dominance, run-time for bounded error, . . .)

- For other search algorithms, “it works well in practice” is often as good an analysis as one gets.
Most texts apply heuristic functions to states, not nodes.

This is slightly less general than our definition:
- Given a state heuristic $h$, we can define an equivalent node heuristic as $h'(\sigma) := h(state(\sigma))$.
- The opposite is not possible. (Why not?)

There is good justification for only allowing state-defined heuristics: why should the estimated distance to the goal depend on how we ended up in a given state $s$?

We call heuristics which don’t just depend on $state(\sigma)$ pseudo-heuristics.

In practice there are sometimes good reasons to have the heuristic value depend on the generating path of $\sigma$ (e.g., the landmark pseudo-heuristic, Richter et al. 2008).
Let $\Sigma$ be the set of nodes of a given search space.

**Definition (optimal/perfect heuristic)**

The **optimal** or **perfect heuristic** of a search space is the heuristic $h^*$ which maps each search node $\sigma$ to the length of a shortest path from $\text{state}(\sigma)$ to any goal state.

**Note:** $h^*(\sigma) = \infty$ iff no goal state is reachable from $\sigma$. 
Properties of heuristics

A heuristic $h$ is called

- **safe** if $h^*(\sigma) = \infty$ for all $\sigma \in \Sigma$ with $h(\sigma) = \infty$
- **goal-aware** if $h(\sigma) = 0$ for all goal nodes $\sigma \in \Sigma$
- **admissible** if $h(\sigma) \leq h^*(\sigma)$ for all nodes $\sigma \in \Sigma$
- **consistent** if $h(\sigma) \leq h(\sigma') + 1$ for all nodes $\sigma, \sigma' \in \Sigma$ such that $\sigma'$ is a successor of $\sigma$

Relationships?
Greedy best-first search

Greedy best-first search (with duplicate detection)

\[
\begin{align*}
open & := \textbf{new} \text{ min-heap ordered by } (\sigma \mapsto h(\sigma)) \\
open.\text{insert}(\text{make-root-node}(\text{init}())) \\
closed & := \emptyset \\
\textbf{while not} \ open.\text{empty}(): \\
& \quad \sigma = open.\text{pop-min}() \\
& \quad \textbf{if} \ state(\sigma) \notin closed: \\
& \quad \quad \textbf{closed} := \text{closed} \cup \{\text{state}(\sigma)\} \\
& \quad \quad \textbf{if} \ \text{is-goal}(\text{state}(\sigma)): \\
& \quad \quad \quad \textbf{return} \ \text{extract-solution}(\sigma) \\
& \quad \quad \textbf{for} \ \textbf{each} \ \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)): \\
& \quad \quad \quad \sigma' := \text{make-node}(\sigma, o, s) \\
& \quad \quad \quad \textbf{if} \ h(\sigma') < \infty: \\
& \quad \quad \quad \quad open.\text{insert}(\sigma') \\
\textbf{return} \ \text{unsolvable}
\end{align*}
\]
Properties of greedy best-first search

- one of the three most commonly used algorithms for satisficing planning
- **complete** for safe heuristics (due to duplicate detection)
- **suboptimal** unless $h$ satisfies some very strong assumptions (similar to being perfect)
- invariant under all strictly monotonic transformations of $h$ (e.g., scaling with a positive constant or adding a constant)
A* (with duplicate detection and reopening)

\( \text{open} := \textbf{new} \ \text{min-heap ordered by} \ (\sigma \mapsto g(\sigma) + h(\sigma)) \)

\( \text{open.insert(make-root-node(init()))} \)

\( \text{closed} := \emptyset \)

\( \text{distance} := \emptyset \)

\( \textbf{while not} \ \text{open.empty():} \)

\( \quad \sigma = \text{open.pop-min()} \)

\( \quad \textbf{if} \ \text{state}(\sigma) \notin \text{closed} \textbf{ or } g(\sigma) < \text{distance(state}(\sigma)) : \)

\( \quad \quad \text{closed} := \text{closed} \cup \{\text{state}(\sigma)\} \)

\( \quad \quad \text{distance}(\sigma) := g(\sigma) \)

\( \quad \quad \textbf{if} \ \text{is-goal}(\text{state}(\sigma)) : \)

\( \quad \quad \quad \textbf{return} \ \text{extract-solution} (\sigma) \)

\( \quad \quad \textbf{for each} \ (o, s) \in \text{succ(state}(\sigma)) : \)

\( \quad \quad \quad \sigma' := \text{make-node}(\sigma, o, s) \)

\( \quad \quad \quad \textbf{if} \ h(\sigma') < \infty : \)

\( \quad \quad \quad \quad \text{open.insert}(\sigma') \)

\( \textbf{return unsolvable} \)
A* example

Example

0+3
I

3

G

1+2
2+5
5

7

1+3
2+7
8

3+1
4+8
5

2+2
3+5

Systematic search
Heuristics
Heuristic search
Uninformed search
Introduction
AI Planning
M. Helmert
A* example

Example

A* example
A* example

Example

Introduction
Uninformed search
Heuristic search
Heuristics
Systematic search
Local search
A* example

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Example

Diagram with nodes labeled as follows:
- Start node: I
- Nodes with operations: 0+3, 1+3, 1+2, 2+6, 2+7, 2+5, 2+2, 3+1
- Goal node: G

Weights:
- 0+3 = 3
- 1+3 = 4
- 1+2 = 4
- 2+6 = 8
- 2+7 = 9
- 2+5 = 7
- 2+2 = 4
- 3+1 = 4
- 3+5 = 8
- G (Goal) is 0

Path costs:
- From I to 0+3: 3
- From 0+3 to 1+3: 4
- From 1+3 to 1+2: 4
- From 1+2 to 2+6: 8
- From 2+6 to 2+7: 9
- From 2+7 to 2+5: 7
- From 2+5 to 2+2: 4
- From 2+2 to 3+1: 4
- From 3+1 to G: 0

Total cost: 3 + 4 + 4 + 8 + 9 + 7 + 4 + 4 + 0 = 51


**Terminology for A**

- **f value** of a node: defined by \( f(\sigma) := g(\sigma) + h(\sigma) \)
- **generated nodes**: nodes inserted into *open* at some point
- **expanded nodes**: nodes \( \sigma \) popped from *open* for which the test against *closed* and *distance* succeeds
- **reexpanded nodes**: expanded nodes for which \( \text{state}(\sigma) \in \text{closed} \) upon expansion (also called reopened nodes)
Properties of A*

- the most commonly used algorithm for optimal planning
- rarely used for satisficing planning
- **complete** for safe heuristics (even without duplicate detection)
- **optimal** if \( h \) is admissible and/or consistent (even without duplicate detection)
- never reopens nodes if \( h \) is consistent

**Implementation notes:**

- in the heap-ordering procedure, it is considered a good idea to break ties in favour of lower \( h \) values
- can simplify algorithm if we know that we only have to deal with consistent heuristics
- common, hard to spot bug: test membership in *closed* at the wrong time
Weighted A* (with duplicate detection and reopening)

\[ \text{open} := \textbf{new} \ \text{min-heap ordered by } (\sigma \mapsto g(\sigma) + W \cdot h(\sigma)) \\\n\text{open.insert(make-root-node(init()))} \\\n\text{closed} := \emptyset \\\n\text{distance} := \emptyset \\\n\text{while not open.empty():} \]

\[ \begin{align*}
\sigma &= \text{open.pop-min()} \\
\text{if } \text{state}(\sigma) \notin \text{closed} \text{ or } g(\sigma) < \text{distance(state}(\sigma)) : \\
&\quad \text{closed} := \text{closed} \cup \{\text{state}(\sigma)\} \\
&\quad \text{distance}(\sigma) := g(\sigma) \\
&\quad \text{if } \text{is-goal}(\text{state}(\sigma)) : \\
&\qquad \text{return } \text{extract-solution}(\sigma) \\
&\text{for each } \langle o, s \rangle \in \text{succ(state}(\sigma)) : \\
&\quad \sigma' := \text{make-node}(\sigma, o, s) \\
&\qquad \text{if } h(\sigma') < \infty : \\
&\quad\quad \text{open.insert}(\sigma') \\
\end{align*} \]

\text{return unsolvable}
Properties of weighted $A^*$

The weight $W \in \mathbb{R}^+_0$ is a parameter of the algorithm.

- for $W = 0$, behaves like breadth-first search
- for $W = 1$, behaves like $A^*$
- for $W \to \infty$, behaves like greedy best-first search

Properties:

- one of the three most commonly used algorithms for satisficing planning
- for $W > 1$, can prove similar properties to $A^*$, replacing optimal with bounded suboptimal: generated solutions are at most a factor $W$ as long as optimal ones
Hill-climbing

\[ \sigma := \text{make-root-node}(\text{init}()) \]

\textbf{forever:}

\begin{itemize}
  \item \textbf{if} \ is-goal(state(\sigma)):
    \begin{itemize}
      \item \textbf{return} extract-solution(\sigma)
    \end{itemize}
  \end{itemize}

\[ \Sigma' := \{ \text{make-node}(\sigma, o, s) \mid \langle o, s \rangle \in \text{succ}(\text{state}(\sigma)) \} \]

\[ \sigma := \text{an element of } \Sigma' \text{ minimizing } h \text{ (random tie breaking)} \]

- can easily get stuck in \textbf{local minima} where immediate improvements of \( h(\sigma) \) are not possible
- many variations: tie-breaking strategies, restarts
Enforced hill-climbing

Enforced hill-climbing: procedure improve

```python
def improve(σ₀):
    queue := new fifo-queue
    queue.push-back(σ₀)
    closed := ∅
    while not queue.empty():
        σ = queue.pop-front()
        if state(σ) ∉ closed:
            closed := closed ∪ {state(σ)}
            if h(σ) < h(σ₀):
                return σ
            for each ⟨o, s⟩ ∈ succ(state(σ)):
                σ′ := make-node(σ, o, s)
                queue.push-back(σ′)
    fail
```

⇝ breadth-first search for more promising node than σ₀
Enforced hill-climbing (ctd.)

Enforced hill-climbing

\[
\sigma := \text{make-root-node}(\text{init}())
\]

while not \text{is-goal}(\text{state}(\sigma)):

\[
\sigma := \text{improve}(\sigma)
\]

return \text{extract-solution}(\sigma)

- one of the three most commonly used algorithms for satisficing planning
- can fail if procedure improve fails (when the goal is unreachable from \(\sigma_0\))
- complete for \text{undirected} search spaces (where the successor relation is symmetric) if \(h(\sigma) = 0\) for all goal nodes and only for goal nodes