Exercise Sheet 10
Due: January 16, 2009

Exercise 10.1 (Coproducts I)

Show that
(a) \[ [h \circ f, h \circ g] = h \circ [f, g] \]
(b) \((f \amalg g) \circ (k \amalg h) = (f \circ k) \amalg (g \circ h)\)

Exercise 10.2 (Coproducts II)

(a) Let \(A\) and \(B\) be elements of a partially ordered set, considered as a category. What is the coproduct of \(A\) and \(B\)?
(b) Let \(P\) and \(Q\) be objects in \(\text{PoSet}\), the category of partially ordered sets. What is the coproduct of \(P\) and \(Q\)?

Exercise 10.3 (Pushouts)

Consider the following diagram:

\[
\begin{array}{ccc}
\bullet & \longrightarrow & \bullet \\
\downarrow & & \downarrow \\
\bullet & \longrightarrow & \bullet
\end{array}
\]

(a) Prove that if both squares are pushouts, then the outside rectangle (with top and bottom edges the evident composites) is a pushout.
(b) Prove that if the outside rectangle and the left-hand square are pushouts and the whole diagram commutes, then the right-hand square is a pushout.
(c) Try to empirically verify this with some examples in \(\text{Hets}\).

Exercise 10.4 (Pushouts of theories and theory morphisms)

Consider a pushout of theories and theory morphisms in a weakly semi-exact institution:

\[
\begin{array}{ccc}
\bullet & \longrightarrow & \bullet \\
\downarrow & & \downarrow \\
\bullet & \longrightarrow & \bullet
\end{array}
\]

Show that if \(\sigma\) is model-theoretically conservative, then so is \(\sigma'\).

The exercise sheets may and should be worked on in groups of two (2) students. Please write both names on your solution.